MATH 170 - Calculus I - Summer 2017
Chapter 4 Supplement

Our First Economics Lesson

Profit makes the world go 'round.

$$P(x) = R(x) - C(x)$$

- x could be anything, but we're going to consider very basic applications of production of goods, where x stands for the number of products produced.
- $R(x)$ is the total revenue earned by producing and selling x items (usually $R(x) = xp$, where p is the price per item).
- $C(x)$ is the total cost of producing x items (usually $C(x) = s + cx$, where s is some startup cost and c is the cost per item).
- $P(x)$ is the total profit earned by producing and selling x items.

Marginal Cost/Profit/Revenue

"Marginal" cost/profit/revenue is equal to the derivative of the total cost/profit/revenue function.

- Marginal cost $= C'(x) = \dfrac{dC}{dx}$
- Marginal revenue $= R'(x) = \dfrac{dR}{dx}$
- Marginal profit $= P'(x) = \dfrac{dP}{dx}$

Interpreting Marginal Cost/Profit/Revenue.

The cost of producing the x^{th} item is approximately $C'(x-1)$. The revenue earned from selling the x^{th} item is approximately $R'(x-1)$. The profit earned from selling the x^{th} item is approximately $P'(x-1)$.

Example 1. Suppose the total profit (in dollars) from the sale of x DVD's is

$$P(x) = 5x - 0.005x^2 - 450 \qquad 0 \leq x \leq 1000.$$

Then the total profit earned by selling the first 450 DVDs is $P(450) = \$787.50$. The profit earned from the 450^{th} DVD (just the last one, not the total profit from all of them!) is $P'(449) = 5 - 0.01(449) = \$0.51$.

The maximum profit will be the absolute maximum value of the function $P(x)$. To find absolute maximum value (on a closed interval $[0, 1000]$), use our techniques from Section 4.4. How many DVDs need to be sold to maximize profit?

Elasticity of Demand

Definition 1. In a supply/demand model, the elasticity of demand measures the effects of price changes on the value of demand.

$$E \approx \frac{\%\text{ change in demand}}{\%\text{ change in price}}$$

We want change in demand relative to change in price. This is the relative rate of change of $p = D(q)$. (Note that here *price* is in terms of quantity instead of the other way around.)

Using what we know about relative rates of change, this means:

$$E(q) = -\frac{p/q}{dp/dq} = -\frac{D(q)/q}{D'(q)} = -\frac{D(q)}{qD'(q)}.$$

Or, if we want to go back to the model of $q = D(p)$, this is the INVERSE function (switching the role of p and q). So, we can use what we know about derivatives of inverse functions to get...

$$E(p) = -\frac{p/q}{dp/dq} = -\frac{p}{q} \cdot \frac{dq}{dp} = -\frac{p}{D(p)} \cdot D'(p) = -\frac{pD'(p)}{D(p)}$$

Example 2. For $p = D(q) = 400 - q$, find $E(q)$ and the point elasticity of demand at $q = 125$.

Example 3. For $p = D(q) = 400/q$, find $E(q)$ and the point elasticity of demand at $q = 50$.

Example 4. For $p = D(q) = 250e^{-q/50}$, find $E(q)$ and the point elasticity of demand at $q = 50$.

Example 5. The demand equation for a certain product is $q = 500 - 40p - p^2$, where p is the price per unit in dollars and q is the quantity of units demanded in thousands. Find the point elasticity of demand when $p = 15$. If the price is increased by 0.5%, what is the approximate change in demand?

Interpreting Elasticity.

If the price p increases by 1%, then the demand (quantity) will increase by $E(p)$%.

- When $|E| < 1$, we say the demand is inelastic. When demand is inelastic, revenue is increasing. An increase in price would increase revenue. (This is good for business!)
- When $|E| > 1$, we say the demand is elastic. When demand is elastic, revenue is decreasing. An increase in price would decrease revenue. (This is bad for business!)
- When $|E| = 1$, we say the demand has unit elasticity. When demand has unit elasticity, revenue is at a maximum.

Example 6. Find the elasticity of $p = D(q) = 200 - 3q$. Then find the elasticity when $q = 30$. Is the demand elastic, inelastic, or of unit elasticity when $q = 30$?

Example 7. Find the elasticity of $q = D(p) = \dfrac{100}{(p+3)^2}$. Then find the elasticity when $p = 1$. Determine the values of p for which the demand is elastic, inelastic, and of unit elasticity.

Example 8. Find the elasticity of $q = D(p) = (p - 50)^2$. Then find the elasticity when $p = 10$. Is the demand elastic, inelastic, or of unit elasticity when $p = 10$?

Example 9. Find the elasticity of $p = D(q) = \sqrt{200 - q^3}$. Then find the elasticity when $q = 3$. Is the demand elastic, inelastic, or of unit elasticity when $q = 3$? For what value(s) of q is revenue maximized?

Practice On Your Own

1. The price-demand equation for an order of fries at a fast-food restaurant is

$$q + 1000p = 2500.$$

   Currently, the price of an order of fries is \$0.99. If the price is decreased by 10%, will revenue increase or decrease? What price will maximize the revenue from selling fries?

2. The total profit (in dollars) from the sale of x calendars is

$$P(x) = 22x - 0.2x^2 - 400 \qquad 0 \leq x \leq 100.$$

   Use the marginal profit to approximate the profit from the sale of the 41st calendar.

3. The demand function for oven mitts is given by $q = -8p + 80$, where q is the number of oven mitts and p is the price in dollars. Find the elasticity of demand when $p = \$7.50$. Will revenue increase if the owner raises the price from \$7.50?

4. The demand function for a clock salesman is $q = -35p + 205$. Find the elasticity of demand when $p = \$5$. Should the salesman raise or lower the price to increase revenue?

5. For the price-demand equation

$$q = D(p) = 1875 - p^2$$

   determine if the demand is elastic, inelastic, or of unit elasticity at $p = 15$, $p = 25$, and $p = 30$.

6. For the price-demand equation

$$x + 500p = 10000$$

   find the elasticity of demand at $p = 4$, $p = 16$, and $p = 10$.