MATH 170 - Calculus I - Spring 2017
Prerequisite Worksheet

All material on this worksheet is prerequisite material that I assume you have seen before and should be able to do **WITHOUT A CALCUATOR**. If there is any material on this worksheet that you do not recognize, you should seriously consider taking the prerequisites for this course instead of jumping right into calculus.

1. For each of the following numbers, circle **all** of the categories that the number falls into:

   (a) 0.445

   Natural Number Integer Rational Number

   Irrational Number Real Number Complex Number

   (b) $\frac{9}{71}$

   Natural Number Integer Rational Number

   Irrational Number Real Number Complex Number

   (c) 1

   Natural Number Integer Rational Number

   Irrational Number Real Number Complex Number

   (d) $\sqrt{3}$

   Natural Number Integer Rational Number

   Irrational Number Real Number Complex Number

   (e) -16

   Natural Number Integer Rational Number

   Irrational Number Real Number Complex Number

   (f) $\sqrt{-16}$

   Natural Number Integer Rational Number

   Irrational Number Real Number Complex Number

(g) $\frac{1}{3}$

Natural Number Integer Rational Number

Irrational Number Real Number Complex Number

(h) π

Natural Number Integer Rational Number

Irrational Number Real Number Complex Number

(i) $\frac{\sqrt{18}}{2}$

Natural Number Integer Rational Number

Irrational Number Real Number Complex Number

(j) $-\sqrt{25}$

Natural Number Integer Rational Number

Irrational Number Real Number Complex Number

2. For each of the following intervals, (1) draw the interval on a number line AND (2) re-write as an inequality (assume the statements express intervals for the variable x).

   (a) $(-1, 5)$

   (b) $\left[-10, -\frac{4}{3}\right)$

   (c) $(1, 2] \cup [5, 7]$

   (d) $(0, \infty)$

   (e) $(-\infty, 0) \cap [-1, \pi]$

3. Perform the following operations and simplify as much as possible. Your final answer will be a single number or fraction and should not include any negative exponents.

   (a) $\frac{1}{9} + \frac{4}{15}$

   (b) $\frac{6}{7} - \frac{3}{14}$

   (c) $\frac{8}{3} \cdot \frac{9}{10}$

   (d) $\frac{5}{4} \cdot \frac{14}{30}$

(e) $\frac{5}{4} \div \frac{14}{30}$

(f) $9 \cdot \frac{1}{5} + \frac{2}{5} \div 6$

(g) $\frac{\frac{7}{9}}{10}$

(h) $\frac{\frac{3}{5}}{\frac{7}{19}}$

(i) 3^3

(j) $(2^2)^3$

(k) $(-1)^5$

(l) -1^5

(m) $\frac{1}{2} - 1$

(n) $\frac{6}{5} - 1$

(o) $-\frac{6}{7} - 1$

(p) $(-27)^{2/3}$

(q) $64^{-1/2}$

(r) $\left(\frac{4}{9}\right)^{-3/2}$

4. Perform the following operations and simplify as much as possible.
Assume $x > 0, y > 0, z > 0$. Your final answer should not include any negative exponents.

(a) $\left(\frac{1}{3}x^2y^{-1}\right)^{-2}$

(b) $-3(z^3x)^4$

(c) $\frac{(x^2)^{-3}}{x^7}$

(d) $(16x^4y^6)^{-3/2}$

(e) $\frac{15x^{3/2}}{3x^{1/4}}$

5. Evaluate/simplify each expression below. Assume $x > 0, y > 0, z > 0$. Your final answer should not include any negative exponents or decimals. Simplify all square roots (example: rewrite $\sqrt{12}$ as $2\sqrt{3}$ in your final answer).

(a) $\sqrt{32}$

(b) $\sqrt[3]{-8x^3}$

(c) $\sqrt{\dfrac{16x^6}{24z^2}}$

(d) $6\sqrt{5}+\sqrt{5}$

(e) $6\sqrt{5}-\sqrt{5}$

(f) $(6\sqrt{5})(\sqrt{5})$

(g) $\sqrt{45x}+\sqrt{20x}$

(h) Rationalize the numerator: $\dfrac{\sqrt{3+x}-\sqrt{3}}{x}$

(i) Rationalize the denominator: $\dfrac{\sqrt{5}+\sqrt{6}}{\sqrt{5}-\sqrt{6}}$

6. Factor the following polynomials. If a polynomial does not factor, write "irreducible" and be able to explain why you came to the conclusion that it is irreducible. Note that these are not equations so you are not "solving" for anything.

(a) $x^2-7x+10$

(b) x^3-7x^2+x-7

(c) $27-x^3$

(d) $36x^2+12x+1$

(e) $12x^3-33x^2$

(f) x^4-81

(g) $12x^7+4x^5+3x^4+x^2$

(h) $3x^2+9$

(i) $2x^2-10x+13$

(j) $2x^2+7x+3$

(k) $x^2+8x+16$

(l) $20x^4-5$

(m) $54x^2+45x-21$

(n) x^2-121

(o) x^2+2x+5

(p) $7x^3+56$

(q) $2x^3-32x^2-34x$

(r) $8x^2+9$

(s) $4x^2-6x+3$

7. Simplify the expressions below. This means writing each one as a single fraction (if it isn't already) and then combining terms, factoring, and cancelling and/or combining common factors where possible.

   (a) $\dfrac{2x-6}{9-x^2}$

   (b) $\dfrac{3-x}{x+5} \cdot \dfrac{x^2+8x+15}{x^2-4}$

   (c) $\dfrac{x^2+2x-3}{x^2+8x+16} \div \dfrac{x-1}{3x+12}$

   (d) $\dfrac{2x-3}{9x^2-1} + \dfrac{4x-1}{(3x-1)^2}$

   (e) $\dfrac{2}{5+x} + \dfrac{5}{x^2-25} + \dfrac{7}{5-x}$

   (f) $\dfrac{x-\frac{1}{x}}{\frac{1}{x^2}-1}$

8. Consider the function $h(x) = \sqrt{7-x} + \sqrt{x+4} - 3$.

   (a) Find the domain of h.

   (b) Find $h(-2)$ and $h(3)$.

   (c) Find the equation of the line containing the points $(-2, h(-2))$ and $(3, h(3))$.

9. Consider the functions $p(x) = \dfrac{6}{x^2-4}, x \geq 0$ and $q(x) = \dfrac{x-2}{3-x}$.

   (a) What is the domain of p? What is the domain of q?

   (b) Find and simplify an expression for the function $(p \cdot q)(x)$. What is the domain of $(p \cdot q)(x)$?

   (c) Find and simplify an expression for the function $(q \circ p)(x)$?

   (d) What is the domain of $(q \circ p)(x)$?

10. Let $f(x) = 2x^2 + 3$. Compute and simplify $\dfrac{f(a+h) - f(a)}{h}$.

11. If $f(x) = \dfrac{x-6}{x}$ and $g(x) = x^2 + 9$, find $(f \circ g)(-2)$.

12. Fill in the table below with exact answers. You need to be able to compute these without referencing a textbook or using a calculator!

t	$\sin t$	$\cos t$	$\tan t$	-	t	$\sin t$	$\cos t$	$\tan t$
0				-	$\frac{5\pi}{6}$			
$\frac{\pi}{6}$				-	π			
$\frac{\pi}{4}$				-	$\frac{5\pi}{3}$			
$\frac{\pi}{3}$				-	$\frac{11\pi}{6}$			
$\frac{\pi}{2}$				-	$\frac{3\pi}{2}$			
$\frac{2\pi}{3}$				-	$\frac{5\pi}{4}$			

13. Draw the following:

    (a) The graph of $y = \sin x$ on the interval $[0, 2\pi]$.
    Label the x-axis at $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ and label the y-axis at $y = 1, -1$.

    (b) The graph of $y = \cos x$ on the interval $[0, 2\pi]$.
    Label the x-axis at $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ and label the y-axis at $y = 1, -1$.

    (c) The graph of $y = \tan x$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Label the x-axis at $x = -\frac{\pi}{2}, -\frac{\pi}{4}, \frac{\pi}{4}\frac{\pi}{2}$ and label the y-axis at $y = 1, -1$.

14. Compute the following (your answer will be a number).

    (a) $\sin\left(\sin^{-1}\frac{1}{5}\right)$
    (b) $\tan(\tan^{-1} 8)$
    (c) $\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)$
    (d) $\sin^{-1}\left(\sin\left(-\frac{5\pi}{6}\right)\right)$
    (e) $\cos^{-1}\left(\cos\frac{\pi}{3}\right)$
    (f) $\tan^{-1}\left(\cos\frac{2\pi}{3}\right)$
    (g) $\cos(\tan^{-1} 2)$
    (h) $\csc\left(\cos^{-1}\frac{7}{25}\right)$

15. Find **all real solutions** for x in the following equations. (Do NOT just find the solutions in the interval $[0, 2\pi]$.)

    (a) $\cos x + 1 = 0$

    (b) $\sin x - 1 = 0$

    (c) $\sqrt{2}\cos x - 1 = 0$

    (d) $\csc^2 x - 4 = 0$

    (e) $2\sin(3x) + 1 = 0$

    (f) $2\sin\frac{x}{3} - \sqrt{3} = 0$

    (g) $\tan^5 x - 9\tan x = 0$

    (h) $2\cos^2 x - 5\cos x - 3 = 0$

Additional Prerequisite Practice Problems

Problem numbers below refer to the "Exercise Set" portion of each section in the course textbook.

Section 0.1

Problems 1, 3, 4, 5, 6, 9, 10, 25, 26

Section 0.2

Problems 1, 2, 27-34, 53-56

Section 0.3

Problems 13, 14, 17, 18, 31, 32, 33, 35, 36

Section 0.4

Problems 31-41

Section 0.5

Problems 1, 2, 5, 6, 9-38