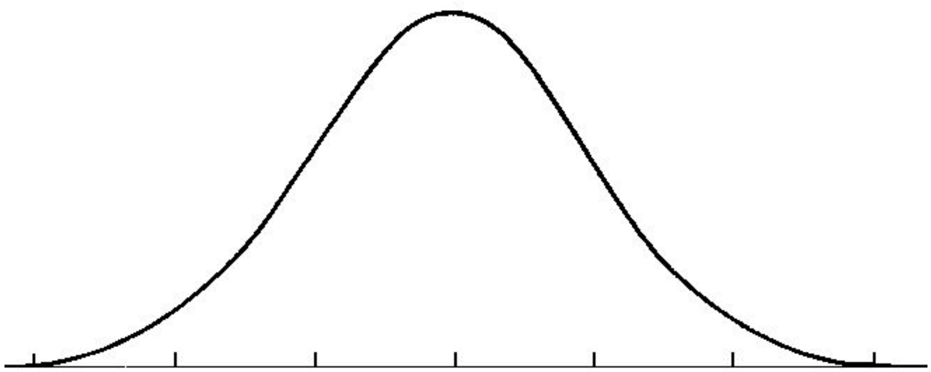
# MATH 109 Lecture Notes

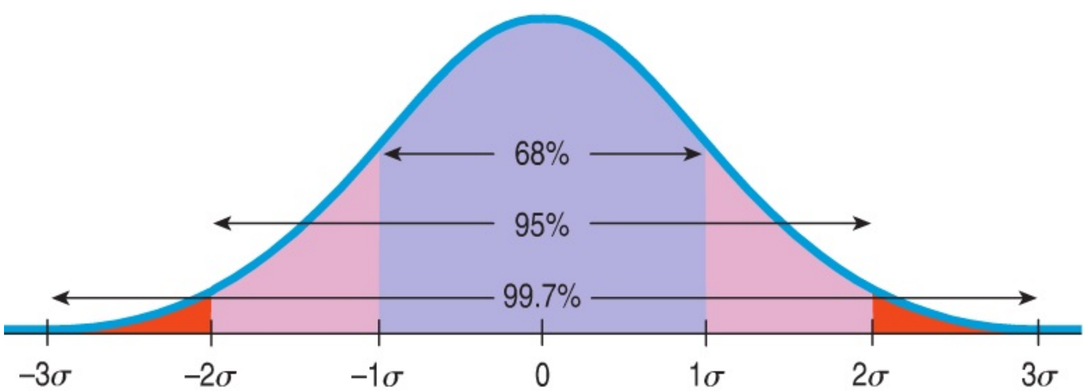
## 5 The Normal Model

### 5.1 Introduction to the Normal Distribution

**Definition 1:** When a distribution is unimodal and close to symmetric, we can use a Normal model to approximate and summarize the data. This model requires that you know the mean and standard deviation of the distribution. When a distribution has mean $\mu$ and standard deviation $\sigma$, the Normal model is denoted by $N(\mu, \sigma)$. With this information, the model will tell us how many values can be expected to fall above, below, or between given values.



**Definition 2:** When we apply the model to the $z$-scores of the distribution of a quantitative variable, it is called the standard Normal model. The $z$-scores of a quantitative variable always have mean 0 and standard deviation 1, so this corresponds to $N(0, 1)$.




**Example 5.1:** Answer the following questions for the standard normal model, $N(0, 1)$.

1. What percentage of the data has a $z$-score greater than 2?

2. What percentage of the data has a $z$-score less than -1?

3. What percentage of the data has a $z$-score less than 1?

4. What percentage of the data has a $z$-score between -1 and 2?

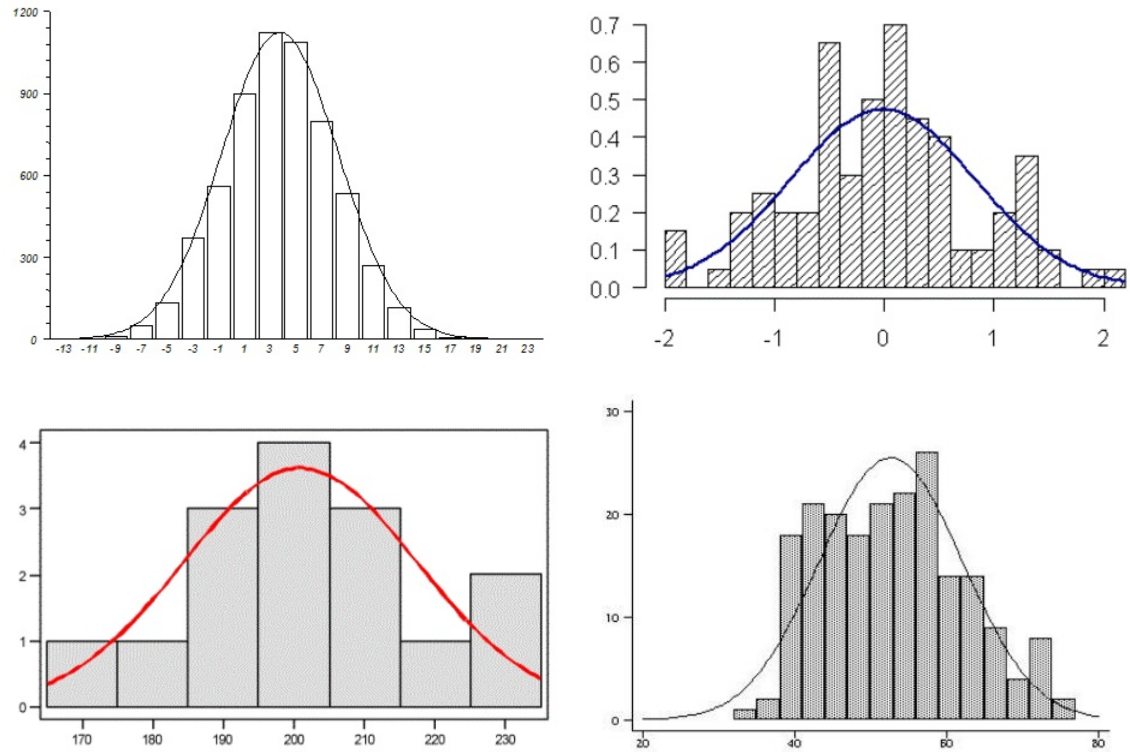
5. What percentage of the data has a $z$-score between 1 and 3?

6. What percentage of the data has a $z$-score greater than -3?

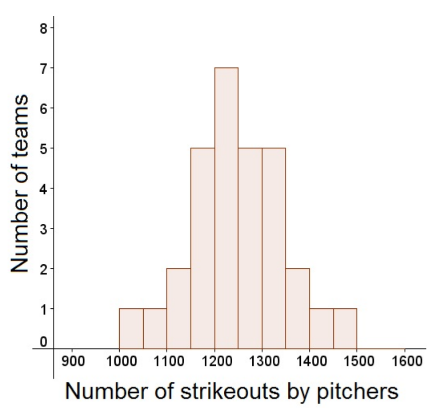
**Distributions vs. Models** Before we see the power of the Normal model in action, let's take a moment to discuss what the difference is between a distribution and a model.

| Distribution | Model |
|---|---|
| Real data | Theoretical values and approximations |
| Observations | Imaginings and "guesses" |
| Histogram | Mathematical curve |
| Calcculated Statistics | Parameters |
| Mean $\overline{y}$ | Mean $\mu$ |
| Std. Dev. $s$ | Std. Dev. $\sigma$ |

**Example 5.2:** For which of the following histograms does the Normal model appear to be a reasonable approximation?



**Example 5.3:** Consider the histogram below showing the number of strikeouts against each MLB team's pitching in the 2014 regular season. Judging by the histogram, would you say that the distribution is nearly normal?



**Example 5.4:** Use the graph of the standard normal model, $N(0, 1)$, to answer the questions below.

1. Find the area to the left of the values below:
   (a) $z = 1.32$

   (b) $z = -0.48$

2. Find the area to the right of the values below:
   (a) $z = 0.89$

   (b) $z = -3.11$

   (c) $z = 1.73$

3. Find the area between the values below:
   (a) $z = -1.05$ and $z = -0.14$

   (b) $z = -2.81$ and $z = 1.30$

   (c) $z = 0.62$ and $z = 1.47$

**Example 5.5:** Assume the data for SAT math section scores can be approximated with the Normal model $N(513, 120)$ for the high school graduating class of 2014. Answer the following questions. Draw pictures to help you answer each question.

1. What $z$-score corresponds to a score of 550 points? 400 points? 730 points?

2. What percentage of students would be expected to score above 550 points? between 400 and 550 points? below 730 points?

3. What exam score corresponds to a $z$-score of -1.32? a $z$-score of 2.71?

4. Which exam scores fall within one standard deviation of the mean? What percentage of test-takers are expected to fall within this range?

## 5.3 Normal Distributions: Finding Values

**Example 5.6:** Find the $z$ score such that the area under the standard normal curve to the left of $z$ is equal to:

1. 0.29

2. 0.72

3. 0.01

4. 0.50

**Example 5.7:** Find the $z$ score such that the area under the standard normal curve to the right of $z$ is equal to:

1. 0.05

2. 0.86

3. 0.37

**Example 5.8:** Assume the data for SAT math section scores can be approximated with the Normal model $N(513, 120)$ for the high school graduating class of 2014. Answer the following questions. Draw pictures to help you answer each question.

1. What exam score corresponds to the 80th percentile? the 43rd percentile?

2. What is the IQR for this model (in points scored)?