Today we will study (Part I)

- Sampling distribution of a statistic, which is the distribution of all values of that statistic when all possible samples of the same size are taken from the same population.
- How to find sampling distribution of sample means.
- How to interpret the Central Limit Theorem.
- How to apply the Central Limit Theorem to find the probability of a sample mean

The **sampling distribution of a statistic** is the probability distribution of all values of the statistic when all possible samples of the same size \( n \) are taken from the same population.

### Sampling Distribution of the Mean

**DEFINITION**
The sampling distribution of the mean is the probability distribution of sample means, with all samples having the same samples size \( n \) taken from the same population.

**Properties of Sampling Distributions of Sample Means**

1. The mean of the sample means \( \mu_x \) is equal to the population mean \( \mu \).
   \[
   \mu_x = \mu
   \]
2. The standard deviation of the sample means \( \sigma_x \) is equal to the population standard deviation \( \sigma \) divided by the square root of \( n \).
   \[
   \sigma_x = \frac{\sigma}{\sqrt{n}}
   \]
   \( \sigma_x \) is called the **standard error of the mean**.

Remarks

- The sample mean \( \bar{x} \) varies from sample to sample and is a random variable.
- As a random variable, it has a probability distribution, called the sampling distributions of the mean.
- Some sample statistics, such as \( \bar{x} \) (mean), \( \hat{p} \) (proportion) are good for estimating values of population parameters, whereas others such as the median do not.
- We will see that using sampling distributions one may use sample sizes which represent a tiny percentage of the actual population, and still estimate the population parameters with reasonable accuracy.
- **When working with an individual value from a normally distributed population, use the methods of sections 5.2 and 5.3**
  \[
  z = \frac{x - \mu}{\sigma}
  \]
- **When working with a mean for some sample, be sure to use the value** \( \sigma_x = \frac{\sigma}{\sqrt{n}} \) for the standard deviation of the sample means, then use:
  \[
  z = \frac{\bar{x} - \mu}{\sigma_x} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}
  \]

The Central Limit Theorem

The Central Limit Theorem forms the foundation for the inferential branch of statistics.

<table>
<thead>
<tr>
<th>The Central Limit Theorem and the Sampling Distribution of $\bar{x}$</th>
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<tbody>
<tr>
<td><strong>Given:</strong></td>
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<tr>
<td>1. The random variable $x$ has a distribution (which may or may not be normal) with mean $\mu$ and standard deviation $\sigma$.</td>
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<tr>
<td>2. Simple random samples all of the same size $n$ are selected from the population. (The samples are selected so that all possible samples of size $n$ have the same chance of being selected.)</td>
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<tr>
<td><strong>Conclusions:</strong></td>
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<tr>
<td>1. The distribution of sample means $\bar{x}$ will, as the sample size increases, approach a normal distribution.</td>
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<td>2. The mean of all sample means is the population mean $\mu$.</td>
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<td>$\mu_{\bar{x}} = \mu$  Mean</td>
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<td>3. The standard deviation of all sample means is $\frac{\sigma}{\sqrt{n}}$.</td>
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<tr>
<td>$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$  Standard Deviation</td>
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<tr>
<td>The standard deviation of the sampling distribution of the sample means, $\sigma_{\bar{x}}$ is also called the <strong>standard error of the mean</strong>.</td>
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<tr>
<td><strong>Implementation:</strong></td>
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<td>1. If the original population is not itself normally distributed, then (in general) for samples of size $n$ greater than 30, the distribution of the sample means can be approximated reasonably well by a normal distribution. The approximation gets better as the sample size $n$ becomes larger.</td>
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<tr>
<td>2. If the original population is itself normally distributed, then the sample means will be normally distributed for any sample size $n$ (not just the values of $n$ larger than 30).</td>
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</table>

**Probability and the Central Limit Theorem**

To find the probability that a sample mean $\bar{x}$ will lie within a given interval of the sampling distribution, you must first transform $\bar{x}$ to a $z$-score.

$$ z = \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} $$  Round to the nearest hundredth

**Remarks**

- Note distribution of sample means has the same center as the population, but it is not as spread out (smaller standard deviation).
- The Central Limit Theorem can also be used to investigate rare occurrences. A rare occurrence is one that occurs with a probability of less than 5%.  

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