Derivative Problems

1) $y = (x^2+4x+6)^5$

$$\frac{dy}{dx} = 5(x^2+4x+6)^4(2x+4) = 10(x^2+4x+6)^4(x+2)$$

2) $f(t) = \dfrac{1}{(t^2-2t-5)^4} = (t^2-2t-5)^{-4}$

$$f'(t) = -4(t^2-2t-5)^{-5}(2t-2) = \frac{8(1-t)}{(t^2-2t-5)^5}$$

3) $h(t) = \left(t - \frac{1}{t}\right)^{3/2} = (t - t^{-1})^{3/2}$

$$h'(t) = \frac{3}{2}(t-t^{-1})^{1/2}(1+t^{-2})$$

4) $y = \dfrac{1}{\sqrt[5]{x^2}} = x^{-2/5} \implies \dfrac{dy}{dx} = -\dfrac{2}{5}x^{-7/5}$

5) $G(x) = (3x-2)^{10}(5x^2-x+1)^{12}$

$$G'(x) = 10(3x-2)^9(3)(5x^2-x+1)^{12} + (3x-2)^{10}12(5x^2-x+1)^{11}(10x-1)$$
$$= (3x-2)^9(5x^2-x+1)^{11}\big(30(5x^2-x+1) + 12(3x-2)(10x-1)\big)$$
$$= 6(3x-2)^9(5x^2-x+1)^{11}(85x^2-51x+9)$$

6) $y = (2x-4)^4(8x^2-4)^{-3}$

$$\frac{dy}{dx} = 4(2x-4)(2)(8x^2-4)^{-3} + (2x-4)^4(-3)(8x^2-4)^{-4}(16x)$$
$$= 8(2x-4)(8x^2-4)^{-3} - 48x(2x-4)^4(8x^2-4)^{-4}$$

7) $y = (x^2+1)(x^2+2)^{1/3}$ $\qquad \dfrac{dy}{dx} = 2x(x^2+2)^{1/3} + \dfrac{2x}{3}(x^2+1)(x^2+2)^{-2/3}$

8) $y = \sec^2 x + \tan^2 x$

$$\frac{dy}{dx} = 2\sec x(\sec x\tan x) + 2\tan x\sec^2 x = 4\sec^2 x\tan x$$

9) $R(y) = \dfrac{y^2-1}{(3y+1)^2}$

$$R'(y) = \frac{2y(3y+1)^2 - 2(y^2-1)(3y+1)(3)}{(3y+1)^4} = \frac{2(3y+1)\left[y(3y+1) - 3(y^2-1)\right]}{(3y+1)^4}$$
$$= \frac{2(y+3)}{(3y+1)^3}$$

10) $f(x) = (3x^2)(4x)^{1/2} = 3x^2(2x^{\frac{1}{2}}) = 6x^{5/2}$

$f'(x) = 15x^{3/2}$

11) $g(x) = \frac{2}{x^4 - x^2 + 1} = 2(x^4 - x^2 + 1)^{-1}$ $g'(x) = -2(x^4 - x^2 + 1)^{-2}(4x^3 - 2x)$

12) $h(x) = (3x^2 - 2x)^{4/5}$ $h'(x) = \frac{4}{5}(3x^2 - 2x)^{-1/5}(6x - 2) = \frac{8(3x-1)}{5(3x^2 - 2x)^{1/5}}$

13) $f(x) = (\sin x \sin 3x)^9$

$f'(x) = 9(\sin x \sin 3x)^8(\cos x \sin 3x + 3\sin x \cos 3x)$

$= 9\cos x \sin^8 x \sin^9(3x) + 27 \sin^9(x)\cos(3x)\sin^8(3x)$

14) $y = \frac{1+\sin x}{x + \cos x}$

$\frac{dy}{dx} = \frac{\cos x(x + \cos x) - (1 + \sin x)(1 - \sin x)}{(x + \cos x)^2} = \frac{x\cos x + \cos^2 x - (1 - \sin^2 x)}{(x + \cos x)^2} = \frac{x\cos x - 1 + 1}{(x + \cos x)^2}$

$= \frac{x\cos x}{(x + \cos x)^2}$

15) $f(x) = \frac{\cos x}{\sin x}(\sin x + \tan x) = \cos x(1 + \frac{1}{\cos x}) = \cos x + 1$

$f'(x) = -\sin x$

16) $g(x) = \frac{2x^4 + 3x^2 - 1}{x^2} = 2x^2 + 3 - x^{-2} \Rightarrow g'(x) = 4x + 2x^{-3}$

17) $s(t) = \left(\frac{t^3+1}{t^3-1}\right)^{1/4}$

$s'(t) = \frac{1}{4}\left(\frac{t^3+1}{t^3-1}\right)^{-3/4} \frac{(3t^2(t^3-1) - 3t^2(t^3+1))}{(t^3-1)^2} = \frac{1}{4}\left(\frac{t^3+1}{t^3-1}\right)^{-3/4} \frac{-6t^2}{(t^3-1)^2} = \frac{-1}{2}\left(\frac{t^3+1}{t^3-1}\right)^{-3/4} \frac{3t^2}{(t^3-1)^2}$

18) $f(z) = (2z - 1)^{-1/5} \Rightarrow f'(z) = -\frac{1}{5}(2z - 1)^{-6/5}(2) = -\frac{2}{5}(2z - 1)^{-6/5}$

19) $h(x) = \frac{x}{\sqrt{7 - 3x}}$

$h'(x) = \frac{(7-3x)^{1/2} - \frac{1}{2}x(7-3x)^{-1/2}(-3)}{7 - 3x} = \frac{7 - 3x + \frac{3}{2}x(\sim)^0}{(7-3x)^{3/2}} = \frac{14 - 3x}{2(7-3x)^{3/2}}$

20) $f(x) = (2x^{3/4} + 5x^{-1/6})^{12}$

$f'(x) = 12(2x^{3/4} + 5x^{-1/6})^{11}\left(\frac{3}{2}x^{-1/4} - \frac{5}{6}x^{-7/6}\right)$

21) $y = \frac{1}{3}\cos(a^3+x^3) \Rightarrow \frac{dy}{dx} = \frac{1}{3}(-\sin(a^3+x^3))(3x^2) = -x^2\sin(a^3+x^3)$

22) $s = \left(\frac{1+t^2}{1-t^2}\right)^7$

$s' = 7\left(\frac{1+t^2}{1-t^2}\right)^6 \frac{2t(1-t^2)-(-2t)(1+t^2)}{(1-t^2)^2} = 7\left(\frac{1+t^2}{1-t^2}\right)^6 \frac{4t}{(1-t^2)^2}$

23) $y = \frac{\tan x - 1}{\sec x} = \frac{\frac{\sin}{\cos x} - 1}{\frac{1}{\cos x}} = \sin x - \cos x \Rightarrow \frac{dy}{dx} = \cos x + \sin x$

24) $f(x) = \frac{\tan^2 x}{\sqrt{\sin^6 x + \sin^4 x \cos^2 x}} = \frac{\tan^2 x}{\sqrt{\sin^4(\sin^2 x + \cos^2 x)}} = \frac{\tan^2 x}{\sqrt{\sin^4 x}} = \left(\frac{\frac{\sin x}{\cos x}}{\sin x}\right)^2 = \sec^2 x$

$f'(x) = 2\sec x(\sec x \tan x) = 2\sec^2 x \tan x$

25) $h(x) = \left(x^2 + (x^2+9)^{\frac{1}{2}}\right)^{1/2}$

$h'(x) = \frac{1}{2}\left(x^2+(x^2+9)^{1/2}\right)^{-1/2}\left(2x + \frac{1}{2}(x^2+9)^{-1/2}(2x)\right)$

$= \frac{1}{2}\left(x^2+(x^2+9)^{1/2}\right)^{-1/2}\left(2x + x(x^2+9)^{-1/2}\right)$

26) $r(t) = \left(\frac{2t+5}{7t-2}\right)^{1/3}$

$r'(t) = \frac{1}{3}\left(\frac{2t+5}{7t-2}\right)^{-2/3} \frac{\overbrace{2(7t-2)-7(2t+5)}^{-39}}{(7t-2)^2} = \left(\frac{2t+5}{7t-2}\right)^{-2/3}\frac{-13}{(7t-2)^2}$

$= \frac{-13}{(2t+5)^{2/3}(7t-2)^{4/3}}$

27) $y = (x + x^{\frac{1}{2}})^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(x+x^{\frac{1}{2}})^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right) = \frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)$

28) $R = \frac{t^{\frac{1}{2}}+1}{t^{\frac{1}{2}}-1}$

$\frac{dR}{dt} = \frac{\frac{1}{2}t^{-1/2}(t^{\frac{1}{2}}-1) - \frac{1}{2}t^{-1/2}(t^{\frac{1}{2}}+1)}{(t^{1/2}-1)^2} = \frac{\frac{1}{2}-\frac{1}{2}t^{-1/2}-\frac{1}{2}-\frac{1}{2}t^{-1/2}}{(t^{1/2}-1)^2} = \frac{-1}{t^{1/2}(t^{1/2}-1)^2}$

29) $f(x) = \left(\frac{\cos(x^2)\tan^2(x^2)}{\sec(x^2)}\right)^3 = \left(\frac{\cos(x^2)\frac{\sin^2(x^2)}{\cos^2(x^2)}}{\frac{1}{\cos(x^2)}}\right)^3 = \left(\sin^2(x^2)\right)^3 = \sin^6(x^2)$

$f'(x) = 6(\sin(x^2))^5\cos(x^2)\,2x$

$= 12x\sin^5(x^2)\cos(x^2)$

30) $y = \dfrac{2x}{(3x^2-4)^{1/3}}$

$\dfrac{dy}{dx} = \dfrac{2(3x^2-4)^{1/3} - 2x(\frac{1}{3})(3x^2-4)^{-2/3}(6x)}{\left((3x^2-4)^{1/3}\right)^2}$

$= \dfrac{2(3x^2-4)^{1/3} - 4x^2(3x^2-4)^{-2/3}}{(3x^2-4)^{2/3}} \cdot \dfrac{(3x^2-4)^{2/3}}{(3x^2-4)^{2/3}}$

$= \dfrac{2(3x^2-4) - 4x^2(\sim)^0}{(3x^2-4)^{4/3}} = \dfrac{2x^2-8}{(3x^2-4)^{4/3}}$

31) $g(x) = \dfrac{2x+1}{x-1}$

$g'(x) = \dfrac{2(x-1)-(2x+1)}{(x-1)^2} = \dfrac{-3}{(x-1)^2} = -3(x-1)^{-2}$

$g''(x) = 6(x-1)^{-3}$

32) $f(y) = \dfrac{y}{\sqrt{1-y^2}}$

$f'(y) = \dfrac{(1-y^2)^{1/2} - \frac{1}{2}y(1-y^2)^{-1/2}(-2y)}{1-y^2}\left(\dfrac{(1-y^2)^{\frac{1}{2}}}{(1-y^2)^{1/2}}\right) = \dfrac{1-y^2+y^2(\sim)^0}{(1-y^2)^{3/2}}$

$= (1-y^2)^{-3/2}$

so $f''(y) = -\frac{3}{2}(1-y^2)^{-5/2}(-2y) = 3y(1-y^2)^{-5/2}$

33) $h(x) = \dfrac{3x}{\sqrt{2x^2+7}}$

$h'(x) = \dfrac{3(2x^2+7)^{1/2} - 3x(\frac{1}{2})(2x^2+7)^{-1/2}(4x)}{2x^2+7}\left(\dfrac{(2x^2+7)^{1/2}}{(2x^2+7)^{1/2}}\right) = \dfrac{3(2x^2+7)^1 - 6x^2(2x^2+7)^0}{(2x^2+7)^{3/2}}$

$= 21(2x^2+7)^{-3/2}$

$h''(x) = 21(\frac{-3}{2})(2x^2+7)^{-5/2}(4x) = -126x(2x^2+7)^{-5/2}$

34) $y = 3(5-2x^2)^{-3/4}$

$\dfrac{dy}{dx} = 3(\frac{-3}{4})(5-2x^2)^{-7/4}(-4x) = \dfrac{9x}{(5-2x^2)^{7/4}}$

$\dfrac{d^2y}{dx^2} = \dfrac{9(5-2x^2)^{7/4} - 9x(\frac{7}{4})(5-2x^2)^{3/4}(-4x)}{\left((5-2x^2)^{7/4}\right)^2} = 9(5-2x^2)^{3/4}\left[\dfrac{(5-2x^2)^{4/4} + 7x^2}{(5-2x^2)^{14/4}}\right]$

$= 9\,\dfrac{5-2x^2+7x^2}{(5-2x^2)^{11/4}} = 9\,\dfrac{5+5x^2}{(5-2x^2)^{11/4}} = 45\,\dfrac{1+x^2}{(5-2x^2)^{11/4}}$

35) (a) $(f+g)'(2) = f'(2) + g'(2) = -1 + 2 = 1$

(b) $(4f)'(2) = 4f'(2) = 4(-1) = -4$

(c) $(fg)'(2) = f'(2)g(2) + f(2)g'(2) = (-1)(-5) + 3(2) = 5 + 6 = 11$

(d) $(ff)'(2) = f'(2)f(2) + f(2)f'(2) = 2f'(2)f(2) = 2(-1)(3) = -6$

(e) $\left(\frac{1}{f+g}\right)'(2) = \frac{0 - (f'(2) + g'(2))}{(f(2)+g(2))^2} = \frac{-(-1+2)}{(3+(-5))^2} = \frac{-1}{(-2)^2} = -\frac{1}{4}$

(f) $\left(\frac{5}{g}\right)'(2) = \frac{0 - 5g'(2)}{(g(2))^2} = -\frac{5(2)}{(-5)^2} = \frac{-10}{25} = \frac{-2}{5}$

36) $h'(x) = f'(g(x))g'(x) \Rightarrow h'(1) = f'(g(1))g'(1) = f'(2)g'(1) = 5(6) = 30$

$H'(x) = g'(f(x))f'(x) \Rightarrow H'(1) = g'(f(1))f'(1) = g'(3)f'(1) = 9(4) = 36$

$F'(x) = f'(f(x))f'(x) \Rightarrow F'(2) = f'(f(2))f'(2) = f'(1)f'(2) = 4(5) = 20$

$G'(x) = g'(g(x))g'(x) \Rightarrow G'(3) = g'(g(3))g'(3) = g'(2)g'(3) = 7(9) = 63$