MODERN ALGEBRA I

MEETING INFORMATION

Meeting Times	MWF 9:00 - 9:50
Location	Hirt 213
Website	math.mercyhurst.edu/∼lwilliams/math280
Prerequisite(s)	Math 150, Math 265

Instructor	Lauren Williams, PhD	
Email	lwilliams@mercyhurst.edu	
Office Phone	(814) 824-2226	
Office	Old Main 404	
Office Hours	Mon	1:00 - 1:50
	Tues	9:30 - 11:00, 3:30 - 4:00
	Wed	1:00 - 1:50
	Thurs	9:00 - 10:00
	Fri	1:00 - 1:50

GRADING

40%	**Midterm Exam Average**
35%	**Homework Average**
25%	**Final Exam**

A	B+	B	C+	C	D+	D
90	87	80	77	70	67	60

IMPORTANT DATES

Aug	**23**	First Class Meeting
	28	Last Day to Add/Drop
Sep	**4**	Labor Day, No Class
Oct	**6**	Midterm Exam I
	12-13	Mid-Semester Break, No Class
	31	Advising Day
Nov	**15**	Midterm Exam II
	17	Last Day to Withdraw
	22-24	Thanksgiving Break, No Class
Dec	**8**	Last Class Meeting
	15	Final Exam 8:00 - 10:00

REQUIRED MATERIALS

We will be using *Contemporary Abstract Algebra*, 8th Edition, by Joseph A. Gallian. An older edition of the text would be fine. No other texts or materials are required. You will not be required to bring the text to class, so an electronic version is acceptable.

COURSE CALENDAR

Aug	**23**	Class Intro, Review
	25	Review: Sets
	28	Review: Properties of Numbers
	30	Review: Modular Arithmetic
Sep	**1**	Review: Logic and Proofs
	4	*Labor Day - No Class*
	6	Review: Linear Algebra
	8	Review: Linear Algebra
	11	Review: Functions, Equivalence Relations
	13	Review: Functions, Equivalence Relations
	15	Binary Operations, Closure
	18	Groups, Properties of Groups
	20	Groups, Properties of Groups
	22	Order
	25	Order
	27	Subgroups
	29	Subgroups
Oct	**2**	Cayley Tables
	4	Review
	6	Midterm I
	9	Centers and Centralizers
	11	Cyclic Groups, Generators
	13	*Mid Semester Break - No Class*
	16	Cyclic Groups, Generators
	18	Permutations
	20	Operations with Permutations
	23	The Symmetric Group
	25	The Alternating Group
	27	Group Homomorphisms
	30	Group Homomorphisms
Nov	**1**	Group Isomorphisms
	3	Group Isomorphisms
	6	Lagrange's Theorem
	8	Internal Direct Products
	10	Fund Thm of Finite Abelian Groups
	13	Review
	15	Midterm I
	17	Fund Thm of Finite Abelian Groups
	20	Rings, Properties of Rings
	22-24	*Thanksgiving - No Class*
	27	Rings, Properties of Rings
	29	Integral Domains
Dec	**1**	Integral Domains
	4	Ideals
	6	Ring Homomorphisms
	8	Review, Last Class Meeting
	15	Final Exam 8 - 10

COURSE DESCRIPTION

This is the first semester of a year long sequence on the study of algebraic structures. Course topics include the properties of numbers, equivalence relations, groups, rings, fields, direct products, homomorphisms and isomorphisms, and the natural development of various number systems.

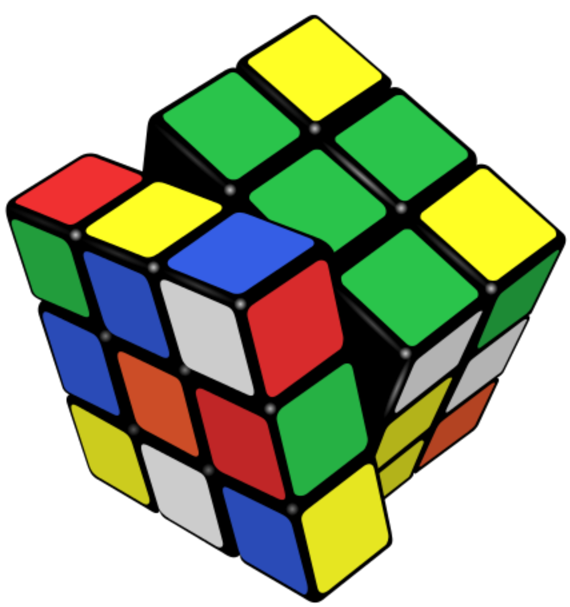
COURSE OBJECTIVES

On successful completion of the course, students should be able to:

- provide the definitions of algebraic objects, and know some examples of each.
- understand the connection between modern algebra and other branches of mathematics.
- relate the material learned in this course to prerequisite courses.
- recognize algebraic structures and objects in everyday situations.
- learn about the historical development of modern algebra.

CLASS POLICIES AND SUGGESTIONS

- Attendance is not required, but is highly recommended. If you have to miss class, read the relevant section of the textbook and try the suggested problems, and ask a classmate for notes and information you may have missed. I do not keep detailed lecture notes for this course.
- I will attempt to return emails as quickly as possible (within 24 hours). However, it is better to ask complicated questions during class or in office hours. If you have a question about the homework, it is quite likely someone else has the same question, so you're doing the class a favor by asking.
- There are other modern algebra textbooks available in my office. Due to book prices, you may not want to invest in a second book, but it can be helpful to have alternate sources or see topics explained in other ways.
- I do not have a "no electronics" policy, but please remember to mute all devices during lecture, and use devices in a way that does not distract other students in the class.
- You will be allowed to listen to music (with headphones) during exams, but please keep the volume at a level that does not distract other students. Plan a playlist in advance - your phone/player will need to be kept face down on the desk throughout the exam.
- While you are encouraged to work together on the homework, be sure you understand all material on your own before an exam.



Depiction of a non-abelian group of order 43,252,003,274,489,856,000

HOMEWORK

You will have several assignment due throughout the semester (generally every Friday). You should expect to spend a fair amount of time on each assignment - don't wait until the night before it's due to get started! You are free to work together on your assignments, but everyone must submit their own work, in their own words. Late homework will be accepted with a 10% per day penalty, until the graded assignments are returned.

Along with each assignment, you will be given a list of suggested problems from the textbook. These are often more computational in nature, and will not be collected. However, these questions may appear on an exam, so please try them and ask questions.

While you are encouraged to come to my office hours or ask clarifying questions about the homework in class, I will not "proof read" your work or grade it until the work is due. It is up to you to decide if you work is correct before turning it in.

EXAMS

We will have two midterm exams and a final exam, as on the course schedule. The final exam will be cumulative, while the midterm exams will focus on more recent material. All exams will be based on homework problems and the suggested textbook problems that do not need to be turned in.

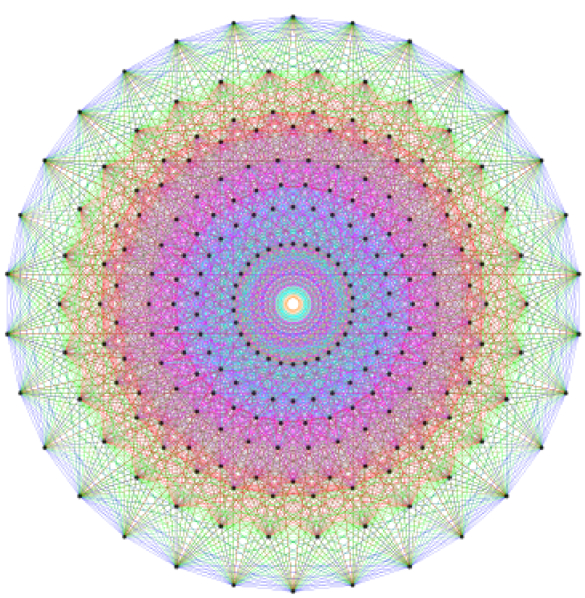
The final exam will be cumulative, and is scheduled for Friday, December 15, 8:00 - 10:00 am.

LEARNING DIFFERENCES

In keeping with college policy, any student with a disability who needs academic accommodations must call Learning Differences Program secretary at 824-3017, to arrange a confidential appointment with the director of the Learning Differences Program during the first week of classes.

MERCY MISSION

This course supports the mission of Mercyhurst University by creating students who are intellectually creative. Students will foster this creativity by: applying critical thinking and qualitative reasoning techniques to new disciplines; developing, analyzing, and synthesizing scientific ideas; and engaging in innovative problem solving strategies.



Mapping of the exceptional simple Lie group E8

ADDITIONAL RESOURCES

Free textbooks on modern algebra, linear algebra, and proofs:

- Joseph Gallian's Abstract Algebra Website
  http://www.d.umn.edu/~jgallian/
  Website of our textbook's author, including some useful links and information.
- *Algebra: Abstract and Concrete*, Frederick M. Goodman
  http://homepage.math.uiowa.edu/~goodman/algebrabook.dir/download.htm
  Free abstract algebra textbook, along with accompanying Mathematica programs and demos from the University of Iowa. Great introduction to symmetry.
- *Abstract Algebra: Theory Applications*, Thomas Judson
  https://open.umn.edu/opentextbooks/BookDetail.aspx?bookId=6
  "...an open-source textbook written by Tom Judson that is designed to teach the principles and theory of abstract algebra to college juniors and seniors in a rigorous manner. Its strengths include a wide range of exercises, both computational and theoretical, plus many nontrivial applications."
- Harvard Extension School's Abstract Algebra Course
  http://www.extension.harvard.edu/open-learning-initiative/abstract-algebra
  Resources for an abstract algebra course, including videos, audio files, and problem sets covering most of the topics we'll be learning. Also includes a section on linear algebra.
- *Book of Proof*, Richard Hammack
  https://open.umn.edu/opentextbooks/BookDetail.aspx?bookId=7
  Free textbook offering a good review of the structure and language of proofs. Also includes a section on relations, functions, and cardinality.
- *Linear Algebra*,Jim Hefferon
  https://open.umn.edu/opentextbooks/BookDetail.aspx?bookId=24
  Linear algebra textbook, if you need a refresher on systems of equations, vector spaces, etc.

Free software

- Wolfram Alpha (Web Application)
  http://www.wolframalpha.com
  Use it to check your work and visualize graphs. From the makers of Mathematica. A (modestly priced) upgrade is available, but the free version allows unlimited computations without an account.
- GAP (Linux, Mac OS, Windows)
  https://www.gap-system.org/
  "GAP is a system for computational discrete algebra, with particular emphasis on Computational Group Theory. GAP provides a programming language, a library of thousands of functions implementing algebraic algorithms written in the GAP language as well as large data libraries of algebraic objects. See also the overview and the description of the mathematical capabilities. GAP is used in research and teaching for studying groups and their representations, rings, vector spaces, algebras, combinatorial structures, and more. The system, including source, is distributed freely. You can study and easily modify or extend it for your special use."
- SageMath (Linux, Mac OS, Web Application)
  http://www.sagemath.org
  An open source mathematics software system. Runs natively on Linux and Mac, but you can also run it within your browser. Plenty of documentation to help offset the learning curve. Based on Python with plenty of useful packages, and you can contribute!

Why Study Modern Algebra?

The following is an excerpt from Joseph Gallian's website on why modern algebra (also known as abstract algebra) is a valuable subject for math and math ed majors:

1. Even though many students take a course is discrete math where they study various proof techniques many of them seem not to absorb this material well. Abstract algebra provides them much more practice at this in a different context than discrete math does.

2. High school math teachers should be very adept at modular arithmetic. Cyclic groups is where they learn this well.

3. Group theory is the mathematics of symmetry–a fundamental notion in science, math and engineering. For example, the symmetry group of a molecule reveals some of its possible (or impossible) chemical properties.

4. There are many important practical applications of modular arithmetic that are best understood by viewing the modular arithmetic in a group theory framework. Examples include the check digits on UPC codes on retail items, ISBN numbers on books, and credit card numbers. In many cases the check digit is the inverse of a weighted sum modulo an integer (10 in the case of a UPC number, 11 in the case of an ISBN number, 9 in the case of Visa travelers checks).

5. Many games can be understood by viewing them as permutation groups. Two examples are the 15 puzzle and the Rubik cube.

6. High school math teachers should be adept at looking at data and making plausible conjectures and generalizations. They should also teach their students to do this. This is a skill that can be learned with practice. Groups and rings provide abundant opportunities for developing this skill.

7. Many people are not comfortable with abstract concepts nor adept at abstract reasoning. The ability to think abstractly is a valuable asset. Abstract algebra helps develop this ability.

8. Abstract Algebra is an ideal capstone course for math ed majors and for those who will go on to grad school in math. Throughout the course they review things like 1-1 functions, onto functions (surprisingly few senior math ed majors understand these ideas well); equivalence relations; basic concepts from linear algebra such as how to multiply matrices, properties of determinants, how to compute a determinant, how to compute the inverse of a matrix, how to tell if a matrix has an inverse, linear transformations (which are group homomorphisms); properties of complex numbers; properties of integers (Euclid's lemma, division algorithm, criterion for divisibility by 9 or 11 or 4); math induction (another important topic that many students do not seem to understand well when they begin an abstract algebra course–this is especially the case for statements that do not involve sums of series); and properties of polynomials (division algorithm, remainder theorem, factor theorem, number of zeros is at most the degree, unique factorization).

9. Doing well in an abstract algebra course is a confidence builder and sometimes causes students to think about going on the graduate school. I once had a student who did extremely well in abstract algebra who went to medical school and now has a high position in the Center for Disease Control in Atlanta. About 20 years after she took the course I met her for dinner while I was at a meeting in Atlanta. I jokingly said to her "Did you use any abstract algebra in med school?" She immediately responded by saying "Abstract algebra was very valuable to me in med school." I asked how. She said that whenever she was taking a difficult course she said to herself "If I can get an A in abstract algebra I can get an A is any course." She was perfectly serious. Many people who start out intending to be high school math teachers or even teach high school math for several years decide to go to grad school in math for an advanced degree (I and many others loved the course and wanted to go to grad school to continue studying the subject). Taking abstract algebra and doing well makes such a move more likely and easier to do.

- Joseph Gallian

HOMEWORK AND STUDYING SUGGESTIONS

Notes on proofs:

1. Remember what the goal of a proof is. You are trying to show that a statement is true, while explaining it so that anyone (with a reasonable level of knowledge) can understand it. A good idea would be to have another student in the class read through your proofs before turning them in. Ask them to point out any statements that is unclear.

2. When proving a statement, be careful to avoid assuming that the statement is true. Keep hypotheses and conclusions separate. It may be a good idea to make a list of facts you can assume (such as hypotheses, definitions, and previous theorems) before you start writing. Make a separate list of the facts you're trying to prove.

3. Proofs should be written using formal language and full sentences. Start your proof with a list of hypotheses you'll be using, and define any symbols or variables you'll be using. You should write a "first draft" of a proof (sometimes several) before writing your final work to turn in. I will focus more on logic/correctness of a proof that on spelling and grammar, but please do your best to write a clear, understandable, and legible proof.

4. Try to stick with conventional notation. Group elements and variables are often represented with lower case letters from the beginning or end of the alphabet (a, b, c, d, g, x, y, z). The letter e is usually reserved for identities. Natural numbers and exponents are often represented with letters from the middle of the alphabet ($k, l, m, n, p, q, r, s, t$). Matrices are denoted with capital letters, and maps, functions, and permutations are given lower case greek letter names (π, ϕ, ψ, τ).

5. It helps to convince yourself that a statement is true before attempting to prove it. Try testing a theorem with a few simple, specific cases, where possible. However, remember that a proof should not include specific examples (unless it's a proof by construction to show an example exists).

6. Use caution with the word "assume" in a proof. Many times, it's better to avoid it entirely. For instance, instead of starting a proof with "assume x is an element of the group G," you could instead write "let x be an element of the group G." However, writing "assume G is Abelian" is probably incorrect, unless it was a hypothesis.

7. Most proofs (in this class) will actually be fairly simple if you're familiar with the definitions. Carefully read the statement, and make sure you understand what each word means and how the definition could be applied.

Notes on homework:

1. Always explain why each conclusion you draw is true. If a question asks something like "What is the order of the group G?" you should supply the answer (a number) along with an explanation of how you arrived at that answer. When writing proofs, unless a statement comes directly from a definition, you should explain why you are able to make it.

2. Until you have more experience with problems and proofs, it's better to write too much than too little. I will point out any inefficiency in your answers, but you will not lose points for providing more information than necessary.

3. A well written proof will almost always have a few drafts before it is finished. The first "draft" may be little more than a few bullet points outlining your argument. Your submitted proofs should be well organized, free of grammar and spelling errors, and clearly written.

4. To make sure your proof is clear and contains enough detail, try asking another student in the class to read it over. If they are not sure why one statement follows from another, it likely means you need to include more supporting information.

Keep in mind that the goal of writing a proof is to construct an irrefutable argument that a statement is true. **If *you* are not convinced that your proof is correct, you cannot assume anyone else will be convinced of it either!**