# MODERN ALGEBRA II

## MEETING INFORMATION

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|---|---|
| **Meeting Times** | MWF 12:00 - 12:50 |
| **Location** | Hirt 313 |
| **Website** | math.mercyhurst.edu/~lwilliams/math281 |
| **Prerequisite(s)** | Math 280 |



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|---|---|
| **Instructor** | Lauren Williams, PhD |
| **Email** | lwilliams@mercyhurst.edu |
| **Office Phone** | (814) 824-2226 |
| **Office** | Old Main 404 |



| **Office Hours** | | |
|---|---|---|
| | Mon | 9:00 - 10:00, 1:00 - 2:00 |
| | Tues | 11:30 - 12:30 |
| | Wed | 11:00 - 11:45 |
| | Thurs | 8:30 - 9:15 |
| | Fri | 9:00 - 11:00 |

## GRADING

| | |
|---|---|
| 30% | **Midterm Exam Average** |
| 30% | **Homework Average** |
| 25% | **Conference Paper and Talk** |
| 15% | **Final Exam** |

| A | B+ | B | C+ | C | D+ | D |
|---|---|---|---|---|---|---|
| 90 | 87 | 80 | 77 | 70 | 67 | 60 |

## IMPORTANT DATES

| | | |
|---|---|---|
| **January** | **17** | First Class Meeting |
| | **19** | Last Day to Add/Drop |
| **February** | **2** | Conference Workshop |
| | **28** | Exam I |
| **March** | **5-11** | Mid-Semester Break - No Class |
| | **16** | Conference Abstract Due |
| | **29** | Easter Break - No Class |
| **April** | **6** | Search for Symmetry |
| | **10** | Advising Day |
| | **11** | Conference Paper Draft Due |
| | **13** | Last Day to Declare Pass/Fail |
| | **18** | Exam II |
| | **20** | Last Day to Withdraw |
| | **27** | Conference Paper Due |
| | **30** | Conference Day 1 |
| **May** | **2** | Conference Day 2 |
| | **4** | Last Class Meeting |
| | **10** | Final Exam 10:30 |

## REQUIRED MATERIALS

We will be using *Contemporary Abstract Algebra*, 8th Edition, by Joseph A. Gallian. An electronic or older version of the text is fine.

## COURSE CALENDAR

| | | |
|---|---|---|
| **Jan** | **17** | Class Intro, Review on Groups |
| | **19** | Cosets |
| | **22** | Cosets |
| | **24** | Lagrange's Theorem |
| | **26** | Normal Subgroups |
| | **29** | Factor Groups |
| | **31** | Factor Groups |
| **Feb** | **2** | Conjugacy Classes |
| | **5** | Conjugacy Classes |
| | **7** | Sylow Subgroups |
| | **9** | Sylow Theorems |
| | **12** | Finite Simple Groups |
| | **14** | Finite Simple Groups |
| | **16** | Finite Simple Groups |
| | **19** | Review on Rings |
| | **21** | Polynomial Rings |
| | **23** | Polynomial Rings |
| | **26** | Review |
| | **28** | Midterm I |
| **Mar** | **2** | Conference Workshop |
| | **5 - 9** | *Spring Break - No Class* |
| | **12** | Division |
| | **14** | Divisibility in Integral Domains |
| | **16** | Divisibility in Integral Domains |
| | **19** | Factorization |
| | **21** | Factorization |
| | **23** | Symmetry |
| | **26** | Symmetries of Planar Objects |
| | **28** | The Rubik's Cube |
| | **30** | *Easter Break - No Class* |
| **Apr** | **2** | *Easter Break - No Class* |
| | **4** | Wallpaper Groups |
| | **6** | Search for Symmetry |
| | **9** | Duality |
| | **11** | Group Actions |
| | **13** | Group Actions |
| | **20** | Review |
| | **18** | Midterm II |
| | **20** | Extension Fields |
| | **23** | Extension Fields |
| | **25** | Intro to Galois Theory |
| | **27** | Intro to Galois Theory |
| | **30** | Conference |
| **May** | **2** | Conference |
| | **4** | Review, Last Class Meeting |
| | **10** | Final Exam 10:30 |

## COURSE DESCRIPTION

This is the second semester of a year long sequence on the study of algebraic structures. Course topics include rings, fields, an introduction to Galois theory, symmetry, the Sylow theorems, and finite simple groups.

The topics for this semester come from four major components:

1. **Additional topics in group theory.** We'll see some new definitions, understand how they can be used to prove important theorems we've already seen, and learn about a few of the major recent advances in algebra.
2. **Divisibility.** A return to rings and polynomials, along with a broader understanding of just what it means to divide, reduce, and factor.
3. **Symmetry.** The many definitions of symmetry (and what they have in common), the symmetry of shapes and patterns, polynomials, and just about any object. This section will naturally revisit the symmetric group and its subgroups. Modern applications to chemistry and physics will have their origin traced back to Leonardo da Vinci, all in the language of abstract algebra.
4. **Galois Theory.** We'll motivate the very existence of modern algebra! This will be an introduction to field extensions, how they arise in the solution of a historical problem, and how the work of Galois continues to influence mathematics.

## COURSE OBJECTIVES

On successful completion of the course, students should be able to:

- provide the definitions of algebraic objects, and know some examples of each.
- develop abstract and critical reasoning by studying and writing mathematical proofs.
- understand the connection between modern algebra and other branches of mathematics.
- relate the material learned in this course to prerequisite courses.
- recognize algebraic structures and objects in everyday situations.
- learn about the historical development of modern algebra.

## CLASS POLICIES AND SUGGESTIONS

- Attendance is not required, but is highly recommended. If you have to miss class, read the relevant section of the textbook and try the suggested problems, and ask a classmate for notes and information you may have missed. I do not keep detailed lecture notes for this course.
- I will attempt to return emails as quickly as possible (within 24 hours). However, it is better to ask complicated questions during class or in office hours. If you have a question about the homework, it is quite likely someone else has the same question, so you're doing the class a favor by asking.
- There are other modern algebra textbooks available in my office. Due to book prices, you may not want to invest in a second book, but it can be helpful to have alternate sources or see topics explained in other ways.
- I do not have a "no electronics" policy, but please remember to mute all devices during lecture, and use devices in a way that does not distract other students in the class.
- You will be allowed to listen to music (with headphones) during exams, but please keep the volume at a level that does not distract other students. Plan a playlist in advance - your phone/player will need to be kept face down on the desk throughout the exam.
- While you are encouraged to work together on the homework, be sure you understand all material on your own before an exam.

## HOMEWORK

As in Math 281, you should expect a homework assignment roughly every Friday. These assignments will be a mixture of "computational" problems and formal proofs. A 10% per day lateness penalty may apply if I do not receive your work on or before the day it is due.

## EXAMS

We will have two midterm exams and a final exam, as on the course schedule. The first midterm exam will cover the additional topics on groups, up through the material on finite simple groups. The second exam will include topics on division and symmetry.

The final exam will be cumulative but will not include any material on field extensions or Galois theory. The final exam is scheduled for Thursday, May 10, 10:30 - 12:30. Please let me know as soon as possible if you will need to reschedule this exam.

## THE SECOND BIENNIAL MATH 281 CONFERENCE

Our class will conclude with a conference, featuring modern algebra. This is your chance to investigate a topic in algebra on your own, and share what you've found with the rest of the class. Some information is provided below, but more detail on expectations will be discussed during the Conference Workshop on Friday, March 2. We'll use this class to discuss possible topics, where to find references, and more.

There will be three major components of your conference grade:

1. **Abstract (10%)**
   Before beginning work on your chosen topic, you'll be required to submit a brief abstract (one to two paragraphs) that describes exactly what you'll be researching and presenting. This need not be detailed, but should give the reader a clear idea of what to expect from your talk and submitted paper.

   The abstract for your talk will be due Friday, March 16.

   If the abstract is "accepted", you will receive full credit for the abstract and may begin work on your seminar talk and write up. If the abstract is not approved, you may revise and resubmit (with no grade penalty) until accepted. The goal will be to choose and describe a topic that is suitably challenging for this level of mathematics, but also possible to explain or demonstrate in a class talk.

2. **Conference Paper (60%)**
   Your conference paper should be detailed narrative of your talk, including proper citations where appropriate. Some guidelines for the conference paper will be provided.

   A draft of your paper will be due Wednesday, April 11. Suggestions for revision will be returned.

   A final paper will be due Friday, April 27.

   The final version of the paper must be typed and include all references. Only the final version will be used for grading. Your papers will be collected and published in the Math 281 Conference Proceedings.

3. **Conference Talk (30%)**
   During the last week of the semester, you'll be sharing your research with your colleagues. Each talk will be twenty minutes, including a few minutes for any questions. While not required, you should make use of the chalkboard, slides, handouts, props, videos, etc to help clarify your presentation.

   You will not be required to submit an outline or plans, but I would be happy to review your strategy or assist in constructing a strong talk.

## SEARCH FOR SYMMETRY

No course in modern algebra would be complete without taking time to appreciate the symmetry all around us. On April 6, we will leave the classroom to look for examples of wallpaper groups, cyclic and dihedral symmetry, and frieze patterns. You will be provided with a ~~field~~ *group* guide to help with your identification.

The search may be postponed for inclement weather. Updates will be made in class.

## SUPPORT OF THE MERCY MISSION

This course supports the mission of Mercyhurst University by creating students who are intellectually creative. Students will foster this creativity by: applying critical thinking and qualitative reasoning techniques to new disciplines; developing, analyzing, and synthesizing scientific ideas; and engaging in innovative problem solving strategies.

## LEARNING DIFFERENCES

In keeping with college policy, any student with a disability who needs academic accommodations must call Learning Differences Program secretary at 824-3017, to arrange a confidential appointment with the director of the Learning Differences Program during the first week of classes.

## ASSESSMENT OF STUDENT LEARNING OUTCOMES

Your work in this course will be used to measure the following learning outcome of the mathematics program:

> *Effectively communicate mathematics, both orally and in writing, with clarity and precision, observing correct notation, syntax, and organization*

This assessment does not affect your grade in any way. Instead, the results will be used to identify any changes we may need to make to improve our program. The rubrics used for this assessment will not be used when grading homework, exams, or your conference materials, but are available on request.