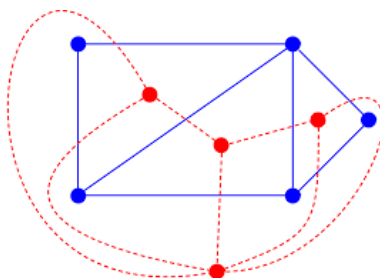


Problem 1: Fill in the chart below with information for the five Platonic solids. The Euler characteristic is the value of $V - E + F$, the number of vertices of the figure, minus the number of edges, plus the number of faces.

| | Face Shape | Vertices, V | Edges, E | Faces, F | Euler Characteristic |
|-----------------------|----------------------|---------------|------------|------------|----------------------|
| Tetrahedron | Equilateral Triangle | 4 | 6 | 4 | |
| Octahedron | | | | | |
| Icosahedron | | | | | |
| Cube (aka Hexahedron) | | | | | |
| Dodecahedron | | | | | |

Problem 2: The dual of a graph can be found by placing a vertex in the center of each face of the graph, then connecting these vertices to form a new graph. For instance, the image below shows a blue graph and its dual in red. Note that the red graph is also the dual of the blue graph.



The same idea can be applied to three dimensional figures. The dual of each of the five Platonic solids is another Platonic solid. Find the “dual pairs”. You may find it helpful to have a model of the solids in front of you.

Problem 3: A plane tiling is a “space filling” pattern, formed by polygons, that lies entirely in two dimensions. The pattern can be repeated infinitely in both directions (length and width). A regular tiling is created entirely out of regular, convex polygons, all of which are the same. An example of such a tiling would be a grid composed of squares. What other regular, convex polygons permit such a tiling? As a hint, the interior angles of the polygons would need to add up to 360° at each vertex, which rules out several of the n -gons. Only one possible tiling can be achieved by each shape, and not all shapes can be tiled.