Math 170 Calculus I

Exam I Review ◦ Exam I: Wednesday, September 19, 2018

Exam I will be based on material from Sections 0.1, 0.2, 0.3, 0.4, 0.5, 1.1, 1.2, 1.3, 1.5 .

You will be expected to know:

- what a **function** is, and how to determine if a graph is a function or just a relation from its graph
- what is meant by the **domain** and **range** of a function, and how to find them
- how to find the composition of two functions
- how to write a function as a composition of two "simpler" functions
- the definitions of polynomial, rational function, exponential function, logarithmic function, and trigonometric function
- how to find the inverse of a function, if it exists, and how to check that your inverse is correct
- the graphs of e^x and $\ln(x)$, and the relationship of these functions
- how to expand a logarithmic expression into a sum of logarithmic expressions that cannot be further simplified
- how to solve logarithmic and exponential equations
- whether the **limit** of a function $f(x)$ exists as x approaches a, ∞, and $-\infty$
- what is meant by a **one-sided limit** and how to decide if a limit exists based on one-sided limits
- how to find a limit, or determine that it does not exist, from a graph
- how to compute a limit at a point for a polynomial or rational function, or decide that it does not exist
- the "limit laws" of Theorem 1.2.2
- how to compute a limit at infinity for a polynomial or rational function, or decide that it does not exist
- how to describe the end behavior of exponential and logarithmic functions
- what it means for a function to be **continuous** at a point and on an interval
- the different types of **discontinuities** that may occur (jump, infinite, removable)
- the properties of continuous functions in Theorem 1.5.3
- that all polynomials are continuous everywhere, as are $\sin(x)$ and $\cos(x)$
- where rational functions are continuous
- that the composition of continuous functions is continuous
- the **Intermediate Value Theorem** (know the theorem and its hypotheses)

Some sample problems:

1. Find the domain of each of the following functions:

   (a) $f(x) = \dfrac{2x+1}{x^2-7x+12}$ (b) $f(x) = \sqrt{x^2-4}$ (c) $f(x) = 8x^3+3x-1$

2. If $y = x^2 + 7x - 2$, find any values of x where $y = -14$.

3. Find $f(0)$, $f(-6)$, and $f(2)$ if $f(x) = \begin{cases} x-3, & x < -2 \\ x^2, & -2 \le x \le 5 \\ 3x+6, & x > 5 \end{cases}$

4. Find $(f \circ g)(x)$ and $(g \circ f)(x)$, and simplify completely, if

   (a) $f(x) = 3x$ and $g(x) = 2x + 5$

   (b) $f(x) = \dfrac{x}{2x+3}$ and $g(x) = \dfrac{1}{x-4}$

   (c) $f(x) = \sqrt{x}$ and $g(x) = x^2 - 8x + 16$

5. If $f(x) = 2(x-1)^3 + 4$, find functions g and h so that $f = g \circ h$.

6. (a) Decide if the function $f(x) = 6x - 8$ has an inverse, and if so, find it.

   (b) Verify that you have found the inverse of $f(x)$.

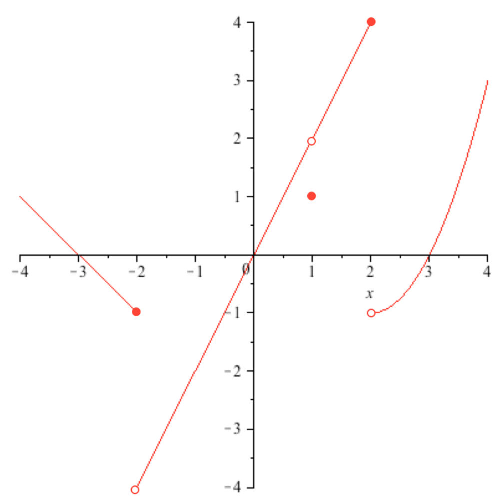
7. Rewrite as a single logarithm: (a) $\frac{1}{3}\log(x) - 4\log(x+1) + \log 100$ (b) $\ln 5 + 3\ln 2 - \ln 16$

8. Solve for x: (a) $\ln(x^3) = 5$ (b) $\log_{10}(1+x) = 2$

9. Expand in terms of sums, differences, and multiples of smaller logarithms that cannot be further reduced.

   (a) $\ln\left(\dfrac{x^5}{\sin^3(x^4)}\right)$ (b) $\ln\left(\dfrac{\sqrt[4]{x-4}}{x^2+1}\right)$

10. Refer to the graph of the function $f(x)$ below:



   (a) Find $f(-2)$, $f(0)$, $f(1)$, and $f(4)$

   (b) Find $\lim\limits_{x\to-2} f(x)$, $\lim\limits_{x\to-2^+} f(x)$, $\lim\limits_{x\to1} f(x)$, $\lim\limits_{x\to0} f(x)$, and $\lim\limits_{x\to2^-} f(x)$

   (c) Find the roots of $f(x)$

11. Find the limits:

   (a) $\lim\limits_{x\to3} \dfrac{4x-12}{x^2-x-6}$ (b) $\lim\limits_{x\to0^+} \dfrac{1}{x^2} + 3$

12. Find the limits:

(a) $\lim_{x\to\infty} \dfrac{7x^2+2x-3}{4x^2+5x+11}$

(b) $\lim_{x\to-\infty} \dfrac{5-2x^3}{x^2+1}$

13. Draw a sketch of a function $f(x)$ with the following properties:

(a) $\lim_{x\to\infty} f(x) = 5$, $\lim_{x\to-\infty} f(x) = -5$

(b) $\lim_{x\to 0} f(x)$ does not exist

(c) $f(0) = 3$

(d) $\lim_{x\to 3} f(x) = 4$

14. Find any values of x where $f(x) = \dfrac{x+3}{x^2-9}$ has a discontinuity, and state the type of discontinuity for each value.

15. Find a value of k so that the function below is continuous:

$$f(x) = \begin{cases} 2x^2-3 & x \geq 4 \\ 6x+k & x < 4 \end{cases}$$

16. Show that the function $f(x) = 3x^2 - 5x - 6$ has a root between -1 and 0. What theorem did you use? Explain why the theorem can be applied to this problem (state the required hypotheses and explain that they are satisfied).