Math 170 Calculus I

Exam III Review ◦ Exam III: Wednesday, November 14, 2018

Exam III will be based on material from Sections 3.3, 3.4, 3.6, 4.1, 4.2, 4.3, 4.4, 4.5.

You will be expected to know:

- the derivatives of exponential functions and compositions involving them
- the derivatives of the six inverse trigonometric functions
- how to set up and solve a related rates problem
- the definition of critical value, and how to find them
- the definition of inflection point, and how to find them
- how to find the intervals on which a function is increasing/decreasing
- how to find the intervals on which a function is concave up/concave down
- how to find the x and y intercepts of a function
- how to find relative extrema and identity whether or not an extrema is a maximum or a minimum
- how to sketch the graph of a polynomial after finding information from its first and second derivatives
- how to sketch the graph of a rational function after finding information from its first and second derivatives
- how to find the absolute extrema, if any, of a function defined on an open, closed, or half-open interval
- how to set up and solve an optimization (applied min/max) problem

Some sample problems:

1. Find $f'(x)$, using any method you prefer:

   (a) $f(x) = \ln(e^x)$

   (b) $f(x) = \dfrac{e^x}{\ln(x)}$

   (c) $f(x) = e^x \sin^{-1}(x)$

   (d) $f(x) = \tan^{-1}(4x^3)$

   (e) $f(x) = \ln\left(\dfrac{1+e^x}{1-e^x}\right)$

   (f) $f(x) = e^{\sqrt{1-3x^2}}$

   (g) $f(x) = \sin^{-1}(x) + \cos^{-1}(x)$

2. Two parallel sides of a rectangle are being lengthened at the rate of 2 in/sec, while the other two sides are shortened in such a way that the figure remains a rectangle with constant area 50 in^2. What is the rate of change of the perimeter of the rectangle when the length of an increasing side is 5 in? Is the perimeter increasing or decreasing?
   NOTE: There will be at least one related rates problem on the exam - it may not be similar to this one, but the more you practice, the better you'll understand the overall strategy.

3. Find the limits:

   (a) $\displaystyle\lim_{x\to+\infty} \frac{x^{60}}{e^x}$

   (b) $\displaystyle\lim_{x\to+\infty} \frac{5x^3-4x+3}{2x^2-1}$

   (c) $\displaystyle\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sin(x)}\right)$

   (d) $\displaystyle\lim_{x\to+\infty} (\sqrt{x^2+x} - x)$

4. For each of the following functions, find all critical values and stationary points. Then, find the intervals on which f is increasing, the intervals on which f is decreasing, the open intervals on which f is concave up, the open intervals on which f is concave down, and the x-coordinates of all inflection points for each of the following:

   (a) $f(x) = x^3 - 3x^2 + x - 2$

   (b) $f(x) = (x-4)^2$

   (c) $f(x) = xe^x$

5. Sketch a graph of the following functions by calculating critical values, inflection points, intercepts, intervals of increasing and decreasing, intervals of concavity, asymptotes, etc.

   (a) $f(x) = x^2 - 3x - 4$

   (b) $f(x) = \dfrac{2x-6}{4-x}$

   (c) $f(x) = \dfrac{3x}{x^2+2x-8}$

6. Find the absolute maximum and absolute minimum, if they exist, of the following functions defined on the given interval.

   (a) $f(x) = x^2$ on the interval $(-\infty, \infty)$

   (b) $f(x) = x^2$ on the interval $[-3, 5]$

   (c) $f(x) = e^x$ on the interval $(0, \infty)$

7. A cylindrical can is being designed to hold 100 cm^3 of oil. The cost of the can depends only on its surface area. Find the dimensions (height and radius) of the can that will minimize the cost of production. (Hint: You need to find an equation for the surface area of a cylinder. The volume of a cylinder with radius r and height h is $V = \pi r^2 h$; the area of a circle of radius r is πr^2, and the perimeter of circle of radius r is $2\pi r$.)