Math 170 Calculus I

Exam II Review ∘ Exam II: Friday, March 2, 2018

Exam II will be based on material from Sections 1.5, 1.6, and 2.1 through 2.6. You are still expected to be familiar with material from earlier sections, but the exam will focus on the homework from these sections.

You will be expected to know:

- what it means for a function to be **continuous** at a point and on an interval
- the different types of **discontinuities** that may occur (jump, infinite, removable)
- the properties of continuous functions in Theorem 1.5.3
- that all polynomials are continuous everywhere, as are $\sin(x)$ and $\cos(x)$
- where rational functions are continuous
- that the composition of continuous functions is continuous
- the **Intermediate Value Theorem** (know the theorem and its hypotheses)
- that if $f(x)$ is continuous and it has an inverse $f^{-1}(x)$, then $f^{-1}(x)$ is continuous
- how to find the tangent line to a function at a point
- the relationship between the tangent line of a function and the derivative
- how to find the derivative of a function using the limit definition (you will need to know the definition - it will not be provided)
- the power rule: $\frac{d}{dx}[x^n] = nx^{n-1}$
- the product rule: $\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
- the quotient rule: $\frac{d}{dx}[\frac{f(x)}{g(x)}] = \frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$
- the chain rule: $\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$
- the derivatives of the six trigonometric functions
- how to find the second derivative of a function and the related notation

Some sample problems:

1. Show that the function $f(x) = 3x^2 - 5x - 6$ has a root between -1 and 0. What theorem did you use? Explain why the theorem can be applied to this problem (state the required hypotheses and explain that they are satisfied).

2. Which of the following functions are continuous everywhere?
   (a) $\sin(x^3 + 2x)$ (b) $3x^2 + \tan(x)$ (c) $\sqrt{(5x+3)}$

3. Find any values of x where $f(x) = \dfrac{x+3}{x^2-9}$ has a discontinuity, and state the type of discontinuity for each value.

4. Find a value of k so that the function below is continuous:
$$f(x) = \begin{cases} 2x^2 - 3 & x \geq 4 \\ 6x + k & x < 4 \end{cases}$$

5. Find all values of x where the tangent line to $f(x) = x^3 - 27x + 18$ is horizontal.

6. Find a point on the graph of $y = x^2$ where the tangent line to y is parallel to the line $y = 6x - 4$.

7. Find $f'(x)$ using the limit definition of the derivative when $f(x) = 3x^2 - 1$. Check your answer using the power rule.

8. Find $f'(x)$ using the limit definition of the derivative.
   (a) $f(x) = \dfrac{1}{x^2}$ (b) $f(x) = \sqrt{2x+1}$ (c) $f(x) = x^3 - 4x$

9. Find $f'(x)$ using any method you prefer:
   (a) $f(x) = \frac{1}{3}(x^6 - 2x^3 + 5)$ (b) $f(x) = 4x^{-2} - 3\sqrt{x}$ (c) $f(x) = \dfrac{4x^5 - 8x^2 + 9}{x^2}$

10. Find $\frac{d^2y}{dx^2}$ if $y = 6x^4 - 9x^2 + 10x - 3$

11. Find $f'(x)$, using any method you prefer:
    (a) $f(x) = \sqrt{2x+1}(5x+2)$
    (b) $f(x) = \dfrac{1 + 4x^3 - 8x^5 + x^7}{x^2}$
    (c) $f(x) = x^6 - 4\sqrt{x} + \dfrac{1}{4x^3}$
    (d) $f(x) = \pi^e - e^\pi + \ln(1)$
    (e) $f(x) = \left(\dfrac{x^2 - 2}{2x^2 + 1}\right)^3$
    (f) $f(x) = \sin(x^3 + 2)$
    (g) $f(x) = \left(\dfrac{\sin(x)}{\cos(x)}\right)^4$
    (h) $f(x) = \dfrac{-2x^3 - 5x}{8x^2 - 10x}$