

Math 170 Calculus I

Exam III Review ◦ Exam III: Friday, April 6, 2018

Exam III will be based on material from Sections 3.1, 3.2, 3.3, 3.4, 3.6, and 4.1. You are still expected to be familiar with material from earlier sections, but the exam will focus on the homework from these sections.

You will be expected to know:

- how to find y' and y'' when y is implicitly defined
- the derivatives of $\ln(x)$ and e^x
- the derivatives of the six inverse trigonometric functions
- how to find the derivative of a function using logarithmic differentiation
- how to set up and solve a related rates problem
- how to find the limit of an indeterminate form of type $\frac{\infty}{\infty}$, $\frac{0}{0}$, $\infty - \infty$, and $0 \cdot \infty$ using l'Hopital's rule
- the definition of critical value, and how to find them
- the definition of inflection point, and how to find them
- how to find the intervals on which a function is increasing/decreasing

Some sample problems:

1. Find $f'(x)$, using any method you prefer:

(a) $f(x) = \ln(x^3 + 4x^2 - 3x + 5)$

(b) $f(x) = \ln(e^x)$

(c) $f(x) = \sqrt{\ln(2x^3)}$

(d) $f(x) = \frac{e^x}{\ln(x)}$

(e) $f(x) = e^x \sin^{-1}(x)$

(f) $f(x) = \tan^{-1}(4x^3)$

(g) $f(x) = \ln\left(\frac{1 + e^x}{1 - e^x}\right)$

(h) $f(x) = e^{\sqrt{1-3x^2}}$

(i) $f(x) = \sin^{-1}(x) + \cos^{-1}(x)$

2. Find y' :

(a) $x^2 + y^2 = 5$

(b) $3xy^2 + 6y - 4x^3 = 10$

(c) $\tan(xy) = y$

3. Find y'' if $x^2 + y^2 = 50$. Simplify your answer as much as possible.

4. Find y' using logarithmic differentiation if

(a) $y = \left(\frac{x^2 - 5}{3x^4 + 6}\right)(x^3 + 1)(3x^4 - 5x^2 + 2x)$

(b) $y = \left(\frac{\sqrt{x}(x^2 - 3)}{\sqrt[3]{5x^3 - 4x + 1}}\right)^4$

5. Two parallel sides of a rectangle are being lengthened at the rate of 2 in/sec, while the other two sides are shortened in such a way that the figure remains a rectangle with constant area 50 in². What is the rate of change of the perimeter of the rectangle when the length of an increasing side is 5 in? Is the perimeter increasing or decreasing?

NOTE: There will be at least one related rates problem on the exam - it may not be similar to this one, but the more you practice, the better you'll understand the overall strategy.

6. Find the limits:

(a) $\lim_{x \rightarrow +\infty} \frac{x^{60}}{e^x}$

(b) $\lim_{x \rightarrow +\infty} \frac{5x^3 - 4x + 3}{2x^2 - 1}$

(c) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\sin(x)}\right)$

(d) $\lim_{x \rightarrow +\infty} (\sqrt{x^2 + x} - x)$

(e) $\lim_{x \rightarrow 0^+} (e^{2x} - 1)^x$

7. For each of the following functions, find all critical values and stationary points. Then, find the intervals on which f is increasing, the intervals on which f is decreasing, the open intervals on which f is concave up, the open intervals on which f is concave down, and the x -coordinates of all inflection points for each of the following:

(a) $f(x) = x^3 - 3x^2 + x - 2$

(b) $f(x) = (x - 4)^2$

(c) $f(x) = xe^x$