Math 170 Calculus I

Exam IV Review ◦ Exam IV: Wednesday, May 2, 2018

Exam IV will be based on material from Sections 4.1, 4.2, 4.3, 4.4, 4.5, 4.8, 5.1, 5.2, and 5.3. You are still expected to be familiar with material from earlier sections, but the exam will focus on the homework from these sections.

You will be expected to know:

- the definition of critical value, and how to find them
- the definition of inflection point, and how to find them
- how to find the intervals on which a function is increasing/decreasing
- how to find relative extrema and identity whether or not an extrema is a maximum or a minimum
- how to find the intervals on which a function is concave up/concave down
- how to find the x and y intercepts of a function
- how to sketch the graph of a function after finding information from its first and second derivatives
- how to find the absolute extrema, if any, of a function defined on an open, closed, or half-open interval
- how to set up and solve an optimization (applied min/max) problem
- the required conditions to apply Rolle's and Mean Value Theorems
- how to find the values predicted by Rolle's and Mean Value Theorems
- how to find the indefinite integral of an expression directly
- how to find the indefinite integral of an expression using substitution
- how to find an exact antiderivative by solving an initial value problem
- the geometric interpretation of the integral
- how to check that an integral is correct

Some sample problems:

1. Find the absolute maximum and minimum of $f(x) = 15x^4 - 15x^2 + 31$ on $[-1, 2]$.

2. Sketch a graph of the following functions by calculating critical values, inflection points, intercepts, intervals of increasing and decreasing, intervals of concavity, asymptotes, etc.

   (a) $f(x) = x^2 - 3x - 4$
   (b) $f(x) = \dfrac{2x-6}{4-x}$
   (c) $f(x) = \dfrac{3x}{x^2+2x-8}$

3. A cylindrical can is being designed to hold 100 cm^3 of oil. The cost of the can depends only on its surface area. Find the dimensions (height and radius) of the can that will minimize the cost of production. (Hint: You need to find an equation for the surface area of a cylinder. The volume of a cylinder with radius r and height h is $V = \pi r^2 h$; the area of a circle of radius r is πr^2, and the perimeter of circle of radius r is $2\pi r$.)

4. Let $f(x) = \frac{1}{2}x - \sqrt{x}$. Verify that the hypotheses of Rolle's Theorem are satisfied on the interval $[0, 4]$, and find any values of c predicted by the theorem.

5. Let $f(x) = x^3 + x - 4$. Verify that the hypotheses of the Mean Value Theorem are satisfied on the interval $[-1, 2]$, and find any values of c predicted by the theorem.

6. Find the integrals

   (a) $\displaystyle\int 4x^2 - 5x^3 + 1 \; dx$
   (b) $\displaystyle\int \sin(x) - \cos(x) \; dx$
   (c) $\displaystyle\int x^4 + x^{-4} \; dx$
   (d) $\displaystyle\int \frac{4x^8 - 2x^4 + 13x^2}{x^3} \; dx$
   (e) $\displaystyle\int \; dx$ (this is not a typo!)
   (f) $\displaystyle\int (x + \sqrt[3]{x})(2 - x^2) \; dx$
   (g) $4\displaystyle\int \sec^2(x) \; dx$
   (h) $\displaystyle\int \frac{5 - 3\sin^2(x)}{\sin^2(x)} \; dx$
   (i) $\displaystyle\int \frac{4}{x^2} \; dx$
   (j) $\displaystyle\int \frac{dx}{1+x^2}$
   (k) $\displaystyle\int \tan(2\theta) \; d\theta$
   (l) $\displaystyle\int x(1 + x^3) \; dx$
   (m) $\displaystyle\int (4x^3 - 6x + 8) \; dx$
   (n) $\displaystyle\int \tan(2\theta) \; d\theta$

7. Find

$$\int_0^2 x + 2 \; dx$$

   geometrically (not by finding an antiderivative).

8. Find the solution $y(x)$ of the initial value problems:

   (a) $y' = 3x^2 - 4, \quad y(0) = 2$
   (b) $y' = 4x^3 - 9 + 2\sin(x) + 7e^x, \quad y(0) = 15$