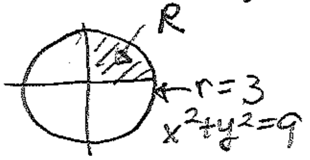
Calculus III Test II March 18, 2014
Dr. Platte Name: KEY

1. (14 points) Use a double integral in polar coordinates to find value of $\iint_R \sqrt{9-x^2-y^2}\,dA$. Where R is the region in the first quadrant within the circle $x^2+y^2=9$.

$z=\sqrt{9-x^2-y^2}$ or $z^2+x^2+y^2=9$, so double integral finds volume of upper half of sphere in first quadrant. ANS $= \frac{4}{3}\pi \cdot 3^3 \cdot \frac{1}{8} = \frac{9}{2}\pi$



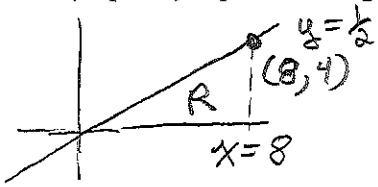
$$V=\int_0^{\frac{\pi}{2}}\int_0^3 \sqrt{9-r^2}\; r\,dr\,d\theta$$

$u=9-r^2$, $-\frac{1}{2}du=r\,dr$
$\int u^{\frac12}\left(-\frac12 du\right) = -\frac12\cdot\frac23 u^{3/2}$

$$V=\int_0^{\frac{\pi}{2}} -\frac13 (9-r^2)^{\frac32}\Big|_0^3\, d\theta$$

$$V = 9\int_0^{\frac{\pi}{2}} d\theta = \frac{9\pi}{2}$$

2. (14 points) Express the integral as an equivalent integral with order of integration reversed.



$$\int_0^4\int_{2y}^8 f(x,y)\,dx\,dy$$

$x=2y$ to $x=8$

$$\int_0^8\int_0^{\frac12 x} f(x,y)\,dy\,dx$$

3. (14 points) Use the chain rule to find $\frac{\partial z}{\partial u}$ *and* $\frac{\partial z}{\partial v}$, where $z = x^2y - 2x + y$ *and* $x = uv, y = u - v$. Leave answer in mixed variable form.

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u}$$

$$\frac{\partial z}{\partial u} = (2xy-2)v + (x^2+1)\cdot 1$$

could write $= 2xyv - 2v + x^2 + 1$

$$\frac{\partial z}{\partial v} = (2xy-2)u + (x^2+1)(-1)$$

could write $= 2xyu - 2u - x^2 - 1$

4. (14 points) Use the total differential (or local linear approximation) to approximate the value of $f(x, y) = \sqrt{1 + xy}$ at point Q(3.99,2.01), knowing the value of f(x, y) at point P (4,2).

$dz = f_x dx + f_y dy$ total diff.

$\therefore$ $f(x_1, y_1) - f(x_0, y_0) \approx f_x(x_0, y_0)(x_1 - x_0) + f_y(x_0, y_0)(y_1 - y_0)$

$(x_1, y_1) = (3.99, 2.01)$, $(x_0, y_0) = (4, 2)$, $f_x = \frac{1}{2} \frac{y}{\sqrt{1+xy}}$, $f_y = \frac{1}{2}\frac{x}{\sqrt{1+xy}}$

$$f(3.99, 2.01) \approx \sqrt{1+4\cdot 2} + \frac{1}{2}\frac{2}{\sqrt{1+4\cdot 2}}(3.99-4) + \frac{1}{2}\frac{4}{\sqrt{1+4\cdot 3}}\cdot(2.01-2)$$

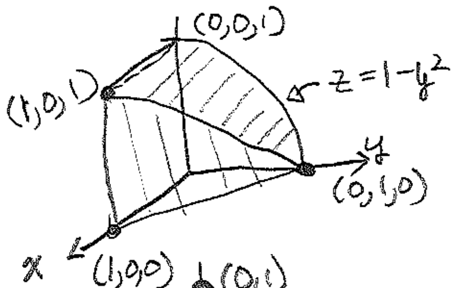
$$\approx 3 - \frac{.01}{3} + \frac{2(.01)}{3} = 3.0033333\ldots$$

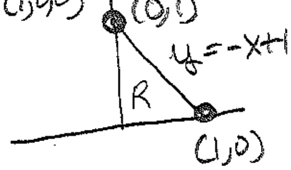
5. (14 points) Use the formula (method for implicit differentiation) developed in this course to find dy/dx for $y^3 + 2x^2y = 3$. Let $F(x,y) = y^3 + 2x^2y$ ie $F(x,y) = 3$

So $\frac{dy}{dx} = -\frac{F_x}{F_y}$, $F_x = 4xy$, $F_y = 3y^2 + 2x^2$

$$\therefore \frac{dy}{dx} = -\frac{4xy}{3y^2 + 2x^2}$$

6. (15 points) Find volume of the solid bounded above by $z = 1 - y^2$, on the bottom by $z = 0$ (xy plane) and on the sides by $x = 0$ (yz plane), $y = 0$ (xz plane) and the plane $y = -x + 1$.





$$V = \int_0^1 \int_0^{-x+1} (1-y^2)\,dy\,dx$$

$$V = \int_0^1 \left(y - \frac{y^3}{3}\right)\Big|_0^{-x+1} dx$$

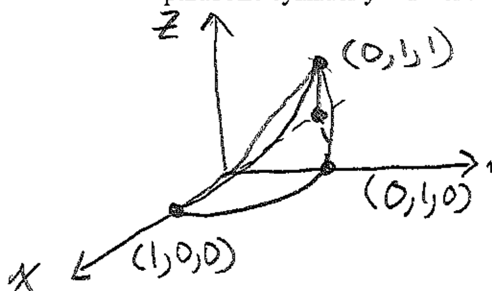
$$V = \int_0^1 \left[(-x+1) - \frac{1}{3}(-x+1)^3\right] dx$$

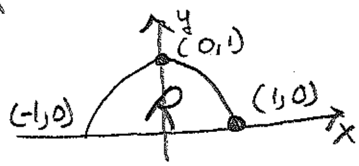
$$V = -\frac{1}{2}(-x+1)^2\Big|_0^1 + \frac{1}{12}(-x+1)^4\Big|_0^1$$

$$V = +\frac{1}{2} - \frac{1}{12} = \frac{5}{12}$$

or $V = \int_0^1 \int_0^{-y+1} (1-y^2)\,dx\,dy$

7. (15 points) Evaluate the triple integral $\iiint_G 1dV$. Where G is the solid enclosed by plane z=y, the xy plane, and the parabolic cylinder $y = 1 - x^2$.





$$\int_{-1}^1 \int_0^{1-x^2} \int_0^y 1\,dz\,dy\,dx$$

$$\int_{-1}^1 \int_0^{1-x^2} y\,dy\,dx$$

$$\int_{-1}^1 \frac{(1-x^2)^2}{2}\,dx$$

$$\frac{1}{2}\int_{-1}^1 [1 - 2x^2 + x^4]\,dx$$

$$\frac{1}{2}\left[x - \frac{2}{3}x^3 + \frac{1}{5}x^5\right]\Big|_{-1}^1$$

$$\left[1 - \frac{2}{3} + \frac{1}{5}\right] = \frac{15-10+3}{15} = \frac{8}{15}$$