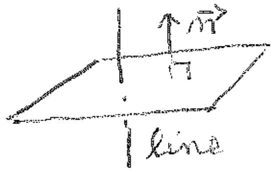
Calculus III | Test III | April 6, 2018
Dr. Platte | | Name:______________________

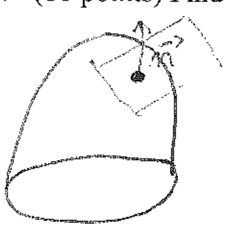
1. (16 points) Find equation of the line perpendicular to the plane $2x + y - 2z = 8$ and through the point (3,2,0).



normal is // to line, $\vec{n} = \langle 2, 1, -2 \rangle$

Eqs $x = 3 + 2t$, $y = 2 + t$, $z = 0 - 2t$

2. (16 points) Find the equation of the tangent plane to the surface $f(x,y) = z = 25 - x^2 - y^2$ at (4, -2).



$\vec{n} = \vec{\nabla} f = -2x\hat{i} - 2y\hat{j} - k$ at $(4,-2)$, $z = 5$

$\vec{\nabla} f = -8\hat{i} + 4\hat{j} - k$

plane $-8(x-4) + 4(y+2) - (z-5) = 0$

$-8x + 4y - z + 45 = 0$

3. (17 points) Find the gradient of f(x,y) at P, and then use the gradient to find the directional derivative at P in the direction of **a**. $z = f(x,y) = x^2y^2 + xy - 5y,\ \ P(2,-1),\ \boldsymbol{a} = -4i + 3j$

$f_x = 2xy^2 + y$ at $(2,-1)$ $f_x = 3$

$f_y = 2x^2y + x - 5$ at $(2,-1)$ $f_y = -11$

$\vec{\nabla} f = 3\hat{i} - 11\hat{j}$

$\|\vec{a}\| = \sqrt{16+9} = 5$, $\vec{u} = -\frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}$

$\vec{\nabla} f \cdot \vec{u} = -\frac{12}{5} - \frac{33}{5} = -9$

4. (17 points) Locate all relative maxima, relative minima, and saddle points, if any, for $f(x,y) = \dfrac{1}{8}x^4 - xy + y^2 + 4$. Be careful, make sure you get all critical points!

Second Partial Test (x_0, y_0) is a critical point. $D = f_{xx}f_{yy} - {f_{xy}}^2$

i. D > 0, $f_{xx} > 0$ then rel. min. at (x_0, y_0)
ii. D > 0, $f_{xx} < 0$ then rel. max. at (x_0, y_0)
iii. D < 0, saddle point at (x_0, y_0)
iv. D = 0, test inconclusive

$f_x = \frac{1}{2}x^3 - y = 0$, $f_y = -x + 2y = 0$, so $x = 2y$, substitute

$\frac{1}{2}(8y^3) - y = 0$, $y(4y^2-1) = 0$, $y(2y-1)(2y+1) = 0$,

$y = 0, \frac{1}{2}, -\frac{1}{2}$; critical pts. $(0,0), (1,\frac{1}{2}), (-1,-\frac{1}{2})$

$f_{xx} = +\frac{3}{2}x^2$, $f_{yy} = 2$, $f_{xy} = -1$

at $(0,0)$
$D = -1$
Saddle pt.

at $(1,\frac{1}{2})$
$f_{xx} = +\frac{3}{2}$
$D = 3-1 = 2$,
$\therefore$ Rel min.

at $(-1,-\frac{1}{2})$
$f_{xx} = \frac{3}{2}$
$D = 3-1 = 2$
$\therefore$ rel min.

$A \quad B \quad C$

5. (17 points) Find equation of the plane containing the points (1,1,1), (2,-2,3) and (2,1,0).

$\vec{AB} = \langle 1,-3,2\rangle$, $\vec{AC} = \langle 1,0,-1\rangle$

$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 1 & -3 & 2 \\ 1 & 0 & -1 \end{vmatrix} = +3i + 3j + 3k = \vec{n}$

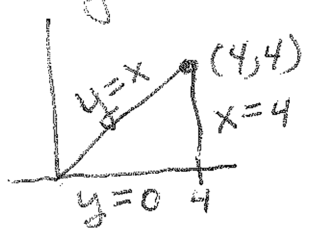
or $n = \langle 1,1,1\rangle$

plane $1(x-1) + 1(y-1) + 1(z-1) = 0$

$1x + 1y + 1z - 3 = 0$

6. (17 points) Find all absoulte maximun and minimum for the function f(x,y) = xy – x - 3y on the closed and bounded triangle with vertices (0,0), (4,0), and (4,4).

Region



Interior

$f_x = y - 1 = 0 \Rightarrow y = 1$

$f_y = x - 3 = 0 \Rightarrow x = 3$

critical pt (3,1)

Boundary $y=0$, $f(x,0) = -x$, decreasing function $\therefore$ critical pts at (0,0) and (4,0).

Boundary $x=4$, $f(4,y) = 4y - 4 - 3y = y - 4$, increasing function. $\therefore$ critical pts (4,0), and (4,4).

Boundary $y = x$, $f(x,y) = x^2 - x - 3y = x^2 - 4x$

$f' = 2x - 4 = 0 \Rightarrow x = 2$. $\therefore$ critical pts (2,2) and (0,0), (4,4).

5 pts	f value
(0,0)	0
(4,0)	-4
(4,4)	0
(3,1)	-3
(2,2)	-4

abs max (0,0) and (4,4)

abs min (4,0) and (2,2)