Chapter 15

($\mathbf{R}^2$ or $\mathbf{R}^3$)   Del operator $\boldsymbol{\nabla} = \frac{\partial}{\partial x}\boldsymbol{i} + \frac{\partial}{\partial y}\boldsymbol{j} + \frac{\partial}{\partial z}\boldsymbol{k}$

Φ(x,y,z) , $\boldsymbol{\nabla\phi}$ is the gradient, a vector field function

$\mathbf{F}(x, y, z) = f(x, y, z)\mathbf{i} + g(x, y, z)\mathbf{j} + h(x, y, z)\mathbf{k} = \mathbf{F}(\mathbf{r})$ Is any vector field function

($\mathbf{R}^2$ or $\mathbf{R}^3$)   **F** is conservative if and only if there exists Φ such that $\boldsymbol{\nabla\phi} = \mathbf{F}$, Φ is a potential function

($\mathbf{R}^3$) $\mathrm{curl}\mathbf{F} = \boldsymbol{\nabla} \times \mathbf{F}$, $\mathrm{div}\mathbf{F} = \boldsymbol{\nabla} \cdot \mathbf{F}$

($\mathbf{R}^2$ or $\mathbf{R}^3$)   Line integrals: , f = f(x,y,z) not vector field function,

$$\int_C f\,ds = \int_a^b f\|\mathbf{r}'(t)\|\,dt, \ C{:}\ \mathbf{r}(t), a \le t \le b,$$

Or **F** a vector field function, $\int_{\boldsymbol{C}} \boldsymbol{F} \cdot d\mathbf{r}$, $d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$

You need parametric equations for the curve. Orientation of curve is important!

($\mathbf{R}^2$)   Fundamental Theorem for line integrals: **F** conservative,

$$\int_{\boldsymbol{C}} \boldsymbol{F} \cdot d\mathbf{r} = \phi(x_1, y_1) - \phi(x_0, y_0)$$

Curve C from (x_0,y_0) to (x_1,y_1). The above implies independence of path as does $\frac{\partial g}{\partial x} = \frac{\partial f}{\partial y}$

($\mathbf{R}^2$)   Green's Theorem: $\oint_c \mathbf{F} \cdot d\mathbf{r} = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA.$

A consequence of Green's Theorem: $\iint_R dA = \frac{1}{2}\oint(-y\,dx + x\,dy)$

($\mathbf{R}^3$)   Stoke's Theorem: $\oint_c \mathbf{F} \cdot d\mathbf{r} = \iint_R \mathrm{curl}\mathbf{F} \cdot \mathbf{n}\,dA$, **F** defined on a surface z = σ(x,y)