1. $z = 2x^2y + 3y^2$, $F(x,y,z) = 2x^2y + 3y^2 - z$

∇F is normal, $F_x = 4xy$, $F_y = 2x^2 + 6y$, $F_z = -1$

$\vec{n}$ at $(2,-1,5) = -8i + 2j - k$

$-8(x-2) + 2(y+1) - (z+5) = 0$ tangent plane

$-8x + 2y - z + 13 = 0$

$\perp$ line $x = 2 - 8t$, $y = -1 + 2t$, $z = 5 - t$

2. $f = x^2y - yz^3 + z$, $f_x = 2xy$, $f_y = x^2 - z^3$, $f_z = -3yz^2 + 1$

$\nabla f|_{(1,1,1)} = 2i + 0j - 2k$

$\vec{u} = \frac{1}{\sqrt{3}}(i - j + k)$. $\vec{\nabla} f \cdot \vec{u} = \frac{2}{\sqrt{3}} - \frac{2}{\sqrt{3}} = 0$

3. $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{4x^2+y^2}$

along x axis, $y = 0$ so $\lim\limits_{x\to 0} \frac{0}{4x^2} = 0$

along $y = x$, $\lim\limits_{x\to 0} \frac{x^2}{4x^2+x^2} = \frac{1}{5}$

limits not equal $\therefore$ no limit.

4. $f(x,y) = x^3 - 3xy + \frac{1}{2}y^2$

$\left.\begin{array}{l} f_x = 3x^2 - 3y \\ f_y = -3x + y \end{array}\right\}$ set to 0 $\quad y = +3x$, $\quad 3x^2 - 3(+3x) = 0$

$3x(x-3) = 0$

$f_{xx} = 6x$, $f_{yy} = 1$, $f_{xy} = -3$ $\quad x = 0$ or $x = 3$

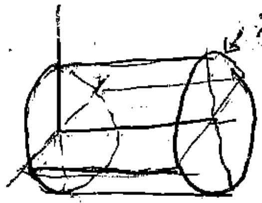
$y = 0$ or $y = 9$

critical pts $(0,0)$ & $(3,9)$

For $(0,0)$ $D = -9$ $\therefore$ saddle pt

For $(3,9)$ $D = 18 - 9 > 0$, $f_{xx} = 18 > 0$ $\therefore$ rel min at $(3,9)$

6.



$x^2+z^2=4$, $z=\sqrt{4-x^2}$ above xy plane

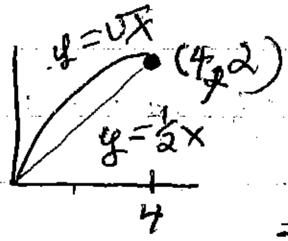
$$\frac{\partial z}{\partial x}=\frac{-2x}{2\sqrt{4-x^2}}$$

Integrand $\sqrt{\frac{x^2}{4-x^2}+1}=\frac{2}{\sqrt{4-x^2}}$

$$\int_0^3\int_{-2}^{2}\frac{2}{\sqrt{4-x^2}}\,dx\,dy=\int_0^3 2\sin^{-1}\frac{x}{2}\Big|_{-2}^{2}=\int_0^3 2\left[\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)\right]dy$$

$$=6\pi$$

7.



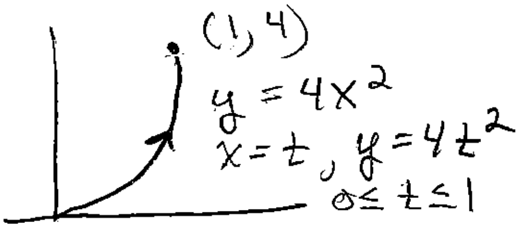
$$\int_0^4\int_{\frac{1}{2}x}^{\sqrt{x}} xy\,dy\,dx$$

$$=\int_0^4 \frac{xy^2}{2}\Big|_{\frac{1}{2}x}^{\sqrt{x}}dx=\frac{1}{2}\int_0^4\left[x^2-\frac{x^3}{4}\right]dx$$

$$=\frac{1}{2}\left[\frac{x^3}{3}-\frac{x^4}{16}\right]_0^4=\frac{1}{2}\left[\frac{64}{3}-\frac{48}{3}\right]=\frac{8}{3}$$

8. $F = x^2 i + xy j$, $dr = dx\, i + dy\, j$

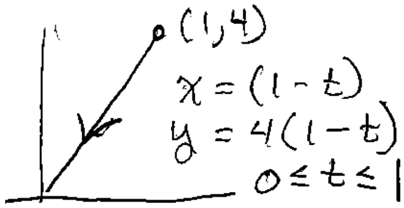
$\int_C F \cdot dr = \int_C x^2 dx + xy\, dy$. But must parameterize the curve! You can not integrate as is!



$dx = dt$ and $dy = 8t\, dt$

Line Integral $\int_0^1 t^2 dt + (t)(4t^2)(8t\,dt)$

$= \int_0^1 (t^2 + 32t^4)\,dt = \left(\frac{t^3}{3} + \frac{32t^5}{5}\right)\Big|_0^1 = \frac{1}{3} + \frac{32}{5} = \frac{101}{15}$.



$dx = -dt$, $dy = -4dt$

Line Integral $\int_0^1 (1-t)^2(-dt) + (1-t)4(1-t)(-4dt)$

$= \int_0^1 -(1-t)^2 dt - 16(1-t)^2 dt = -17\int_0^1 (1-t)^2 dt$

$= -17 \frac{(1-t)^3}{-3}\Big|_0^1 = -\frac{17}{3}$.

Answer: $\frac{101}{15} - \frac{17}{3} = \frac{16}{15}$

9. Maximize $P = xyz$, subject to constraint $x+y+z=48$ and all positive reals.

Let $f(x,y,z) = xyz$ $\quad g(x,y,z) = x+y+z-48$

$\nabla f = yz\,i + xz\,j + xy\,k$, $\nabla g = i + j + k$

$\nabla f = \lambda \nabla g$ so $yz = \lambda = xz = yx$

$yz = xz$ so $x = y$ or $z = 0$

$yz = xy$ so $x = z$ or $y = 0$

So critical points are $(0,0,0)$, $(48,0,0)$, $(0,48,0)$, $(0,0,48)$

or $x = y = z \implies (16,16,16)$.

All the points with a 0 coordinate give abs min

$(16,16,16)$ gives absolute max.

If you do not use LaGrange Multipliers you must be careful in what you set equal.

$P = xyz$, $x+y-48 = -z$

$P = 48xy - x^2y - y^2x$

$\frac{\partial P}{\partial x} = 48y - 2xy - y^2 = 0 \implies y(48-2x-y) = 0$

$\frac{\partial P}{\partial y} = 48x - x^2 - 2xy = 0 \implies x(48-2y-x) = 0$

From 1st eq. either $x = 0$ or $y = 48 - 2x$ *

From 2nd eq. either $y = 0$ or $48 - 2(48-2x) - x = 0$

ie $48 - 96 + 4x - x = 0$ or $3x = 48$, $x = 16$, subst into *

$y = 16$ and $z = 16$.

So either $(0,0,48)$, $(48,0,0)$, $(0,48,0)$, $(0,0,0)$

or $(16,16,16)$.