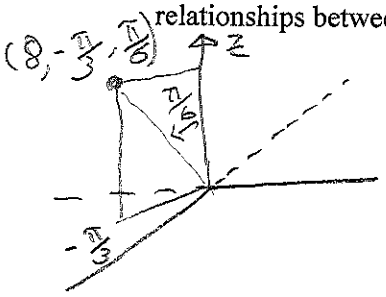
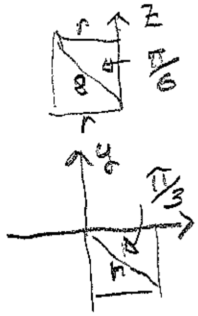
Calculus III
Dr. Platte

Test I
Name: KEY

Feb 18, 2014

1. (12 points) Convert the spherical coordinate point $(8, -\pi/3, \pi/6)$ to both cylindrical and rectangular coordinates. You can use the formulas if you remember them, however plotting the point and using the visual trigonometric relationships between the systems should suffice.





$r = 8 \sin\frac{\pi}{6} = 4$

$z = 8\cos\frac{\pi}{6} = 8\cdot\frac{\sqrt{3}}{2} = 4\sqrt{3}$

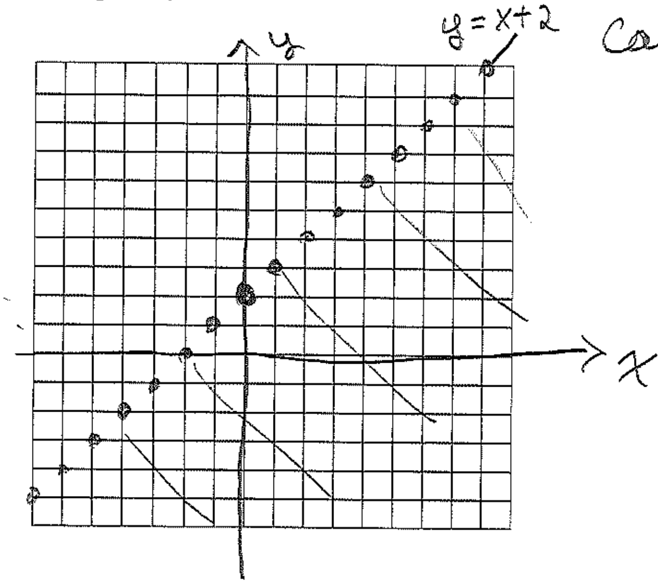
$x = r\cos\frac{\pi}{3} = 4\cdot\frac{1}{2} = 2$

$y = -r\sin\frac{\pi}{3} = -4\,\frac{\sqrt{3}}{2} = -2\sqrt{3}$

rect $(2, -2\sqrt{3}, 4\sqrt{3})$

cyl $(4, -\frac{\pi}{3}, 4\sqrt{3})$

2. (12 points) Find and sketch the natural domain of the function $z = f(x,y) = \ln(x - y + 2)$.



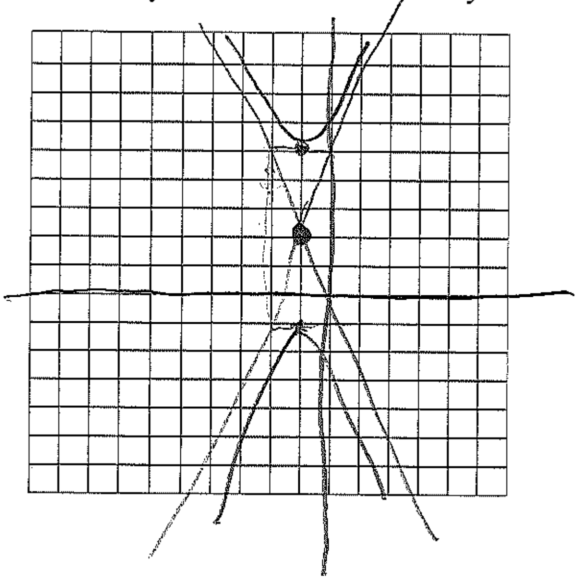
Can't take ln of a negative, or zero

$x - y + 2 > 0$, $x + 2 > y$

graph $y = x + 2$

3. (12 points) Put the conic section given below into standard form by performing appropriate completions of the square. Identify what type of conic section it is and sketch!

$$y^2 - 9x^2 - 18x - 4y - 14 = 0$$



$y^2 - 4y + 4 - 9(x^2 + 2x + 1) = 14 + 4 - 9$

$(y-2)^2 - 9(x+1)^2 = 9$

$\frac{(y-2)^2}{9} - (x+1)^2 = 1$

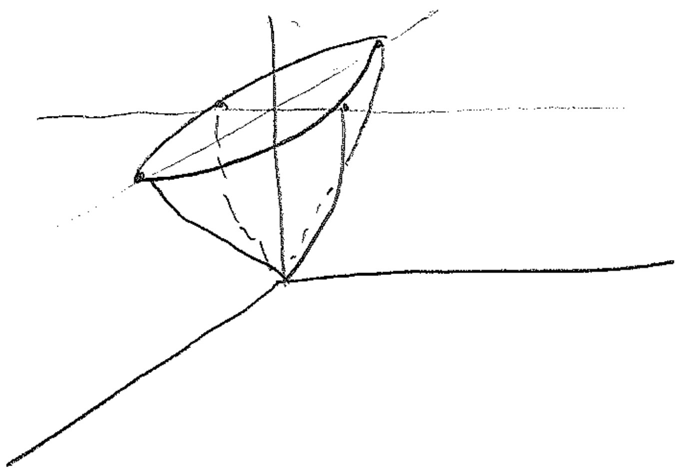
hyperbola center $(-1, 2)$

4. (12 points) Let $z = 4x^2 + 9y^2$ in $\mathbb{R}^3$,identify the conic section you get from the cross sections for z = 36, x = 0 and y = 0. Sketch the surface $z = 4x^2 + 9y^2$ in $\mathbb{R}^3$.

If $z=36$, $36 = 4x^2+9y^2$, $1 = \frac{x^2}{9} + \frac{y^2}{4}$ ellipse

If $x=0$, $z=9y^2$ parabola. If $y=0$, $z=4x^2$ parabola

Note: $z \geq 0$



5. (13 points) Determine whether the following limit exist. If it exists then find its value. If it does not exist explain why!

$$\lim_{(x,y)\to(0.0)} \frac{x+y-x^2y}{x+y}$$

Try paths!

path $x=0$ $\quad \lim_{y\to 0} \frac{y}{y} = 1$

path $y=0$ $\quad \lim_{x\to 0} \frac{x}{x} = 1$

path $y=x$ $\quad \lim_{x\to 0} \frac{2x-x^3}{2x} = \lim_{x\to 0} 1 - \frac{x^2}{2} = 1$

path $y=2x$ $\quad \lim \frac{x+2x-2x^3}{x+2x} = 1$

$\lim \frac{x+y-x^2y}{x+y} = \lim\left(1 - \frac{x^2y}{x+y}\right)$

Along any path, except $y=-x$, the limit would be 1. Function is discont. for any pt on $y=-x$. any disk centered at $(0,0)$ includes pts on line $y=-x$ where funct. is undefined. So no limit exists. Check Definition!

6. (13 points) Find the partial with respect to x and y for the function below. That is find f_x and f_y .

$$f(x,y) = x\ln(xy^2+1)$$

$f_x = \ln(xy^2+1) + x \cdot \frac{1}{xy^2+1} \cdot y^2 = \ln(xy^2+1) + \frac{xy^2}{xy^2+1}$

$f_y = x \cdot \frac{1}{xy^2+1} \cdot 2yx = \frac{2yx^2}{xy^2+1}$

7. (13 points) Use implicit partial differentiation to find $\frac{\partial z}{\partial x}$

xz + xyz² = x

$$\frac{\partial (xz + xyz^2)}{\partial x} = \frac{\partial x}{\partial x}$$

$$z + x\frac{\partial z}{\partial x} + yz^2 + 2xyz\frac{\partial z}{\partial x} = 1$$

$$\frac{\partial z}{\partial x} = \frac{1 - z - yz^2}{x + 2xyz}$$

$$\frac{\partial (xz + xyz^2)}{\partial y} = \frac{\partial x}{\partial y}$$

$$x\frac{\partial z}{\partial y} + xz^2 + 2xyz\frac{\partial z}{\partial y} = 0$$

$$\frac{\partial z}{\partial y} = -\frac{xz^2}{x + 2xyz}$$

$$\frac{\partial z}{\partial y} = -\frac{z^2}{1 + 2yz}$$

8. (13 points) For the polar equation $r = 1 + 3\cos(\theta)$ sketch the proportion of the graph **only** for $0 \le \theta \le \pi$. Fill in the table! Use the provided polar graph paper.

θ	0	π/6	π/4	π/3	π/2	2π/3	3π/4	5π/6	π
r	4	$1+\frac{3\sqrt{3}}{2}$	$1+\frac{3\sqrt{2}}{2}$	$\frac{5}{2}$	1	$-\frac{1}{2}$	$1-\frac{3\sqrt{2}}{2}$	$1-\frac{3\sqrt{3}}{2}$	-2
		3.6	3.12	2.5			-1.12	-1.6	

Note

θ	100°	110°
r	.479	-.026

$1 + 3\cos 109.4° \approx 0$