Calculus III Test I Feb 10, 2017
Dr. Platte Name:______________________

1. (15 points) Determine whether the following limit exist. If it exists then find its value. If it does not exist, explain why!

$$\lim_{(x,y)\to(0,0)} \frac{x^2+xy+y^2}{2x^2+2y^2}$$ Limit does not exist.

Along x axis, ie $y=0$. $\lim_{x\to 0} \frac{x^2+x\cdot 0+0^2}{2x^2+2\cdot 0^2} = \frac{1}{2}$

Along path $y=x$. $\lim_{x\to 0} \frac{x^2+x\cdot x+x^2}{2x^2+2x^2} = \lim_{x\to 0} \frac{3x^2}{4x^2} = 3/4$

Limits not equal thus no limit.

2. (14 points) Find the partial with respect to x and y for the function below. That is find f_x and f_y.

$f(x,y) = x\cos(xy) + y$

f_x = use product rule $= 1\cdot\cos xy + x\frac{\partial \cos(xy)}{\partial x} + 0$

now use chain rule and product rule

$f_x = \cos xy - x\cdot y\sin(xy)$

$f_y = -x^2\sin(xy) + 1$

3. (14 points) For the equation below use implicit partial differentiation to find and solve for $\frac{\partial z}{\partial x}$.

$xz + yz^2 - y = 0$ Z is a function of x and y.

$\frac{\partial xz}{\partial x} = z + x\frac{\partial z}{\partial x}$, $\frac{\partial yz^2}{\partial x} = 2yz\frac{\partial z}{\partial x}$, $\frac{\partial y}{\partial x} = 0$

So $z + x\frac{\partial z}{\partial x} + 2yz\frac{\partial z}{\partial x} = 0$

$$\frac{\partial z}{\partial x} = -\frac{z}{x+2yz}$$

4. (14 points) For the function below find $\frac{\partial^3 f}{\partial x \partial x \partial y} = f_{xxy}$ and $\frac{\partial^3 f}{\partial y \partial y \partial x} = f_{yyx}$ Recall $\frac{d \ln x}{dx} = \frac{1}{x} = x^{-1}$

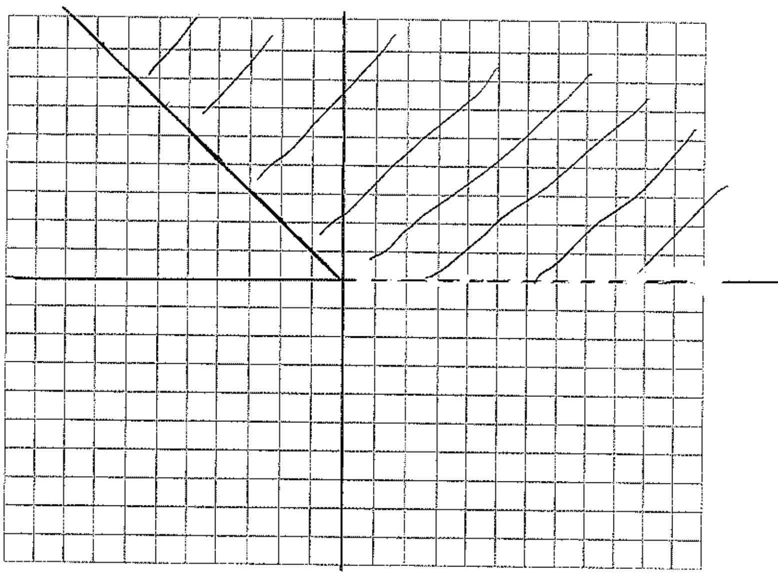
$f(x, y) = xy\ln(y)$

$f_x = y\ln y$ $\qquad f_y = x\ln y + xy \cdot \frac{1}{y} = x\ln y + x$

$f_{xx} = 0$ $\qquad f_{yy} = \frac{x}{y} + 0$

$f_{xxy} = 0$ $\qquad f_{yyx} = \frac{1}{y}$

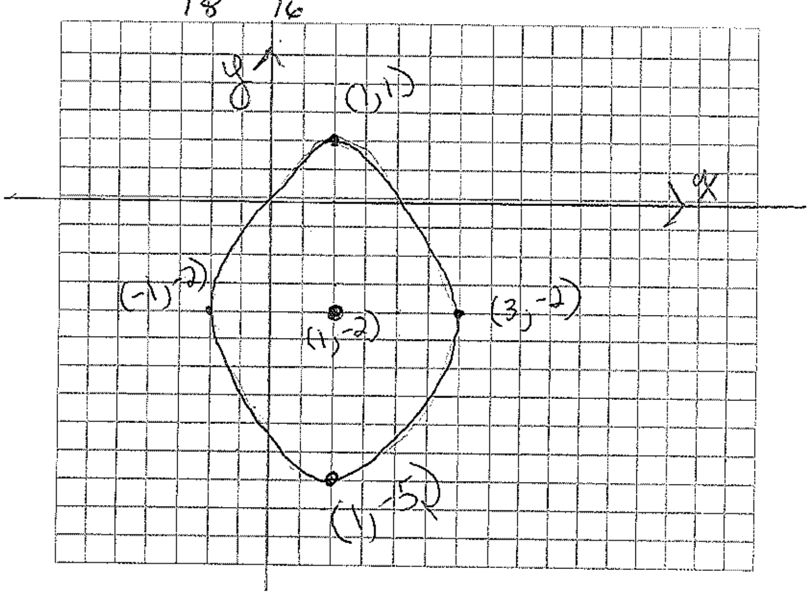
5. (14 points) Find and sketch the natural domain of the function $z = f(x, y) = \sqrt{x + y}\ln(y)$.



$x + y \geq 0$ or $y \geq -x$ and also $y > 0$

6. (14 points) Put the conic section given below into standard form by performing appropriate completions of the square. Identify what type of conic section it is and sketch!

$9x^2 + 4y^2 - \underset{18}{\cancel{16}}x + \underset{16}{\cancel{18}}y = 11$



Rewrite

$9(x^2 - 2x) + 4(y^2 + 4y) = 11$

Now complete squares

$9(x^2 - 2x + 1) + 4(y^2 + 4y + 4) = 11 + 9 + 16$

$9(x-1)^2 + 4(y+2)^2 = 36$

$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$

an ellipse

center $(1, -2)$

7. (15 points) For the polar equation $r = \sin(2\theta)$ sketch the proportion of the graph **only** for $\pi/2 \le \theta \le 3\pi/2$. Fill in the table! Use the provided polar graph paper.

θ	$\pi/2$	$4\pi/6$	$3\pi/4$	$5\pi/6$	π	$7\pi/6$	$5\pi/4$	$8\pi/6$	$3\pi/2$
r	0	-.866	-1	-.866	0	.866	1	.866	0
		$-\frac{\sqrt{3}}{2}$		$-\frac{\sqrt{3}}{2}$		$\frac{\sqrt{3}}{2}$		$\frac{\sqrt{3}}{2}$