Calculus III Test II Feb. 28, 2018
Dr. Platte Name:____

1. (16 points) Use the Calculus III method instead of implicit differentiation to find dy/dx for the equation $x^3y + \cos(xy) = 0$.

$f = x^3y + \cos xy$

$f_x = 3x^2y - y\sin xy$

$f_y = x^3 - x\sin xy$

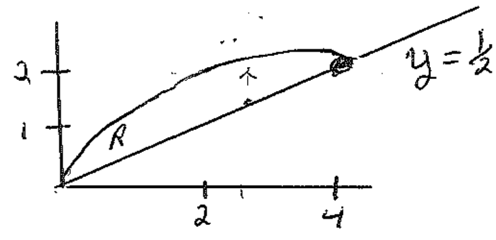
$$\frac{dy}{dx} = -\frac{f_x}{f_y} = -\frac{3x^2y - y\sin xy}{x^3 - x\sin xy}$$

2. (16 points) Let $z = x^2y - xy$, $x = r + r\cos\theta$ and $y = r^2 + r\sin\theta$. Use an appropriate chain rule to find $\frac{\partial z}{\partial \theta}$.
Do not substitute! Leave answer with x, y, r, and θ variables in the results.

$$\frac{\partial z}{\partial \theta} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial \theta} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial \theta}$$

$$\frac{\partial z}{\partial \theta} = (2xy - y)(-r\sin\theta) + (x^2 - x)(r\cos\theta)$$

3. (17 points) Let R be the region in the first quadrant enclosed between $y = \frac{1}{2}x$ and $x = y^2$. Let $f(x,y) = xy$ be the surface over R. Find the enclosed volume.

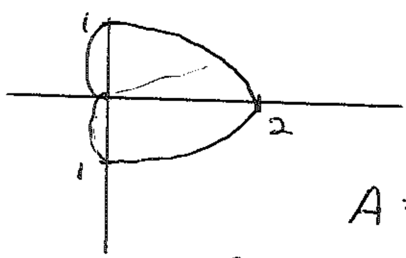


$$\int_0^4 \int_{\frac{1}{2}x}^{\sqrt{x}} xy\,dy\,dx$$

$$= \int_0^4 x\frac{y^2}{2}\Big|_{\frac{1}{2}x}^{\sqrt{x}} dx = \int_0^4 \left[\frac{x^2}{2} - \frac{x^3}{8}\right] dx$$

$$= \left(\frac{x^3}{6} - \frac{x^4}{32}\right)\Big|_0^4 = \frac{32}{3} - 8 = \frac{8}{3}$$

4. (17 points) Use a double integral in polar coordinate to find the area of the portion of the cardioid $r = 1 + \cos\theta$ in the first quadrant.



$$A = \int_0^{\frac{\pi}{2}} \int_0^{1+\cos\theta} 1 \cdot r\,dr\,d\theta$$

$$A = \frac{1}{2}\int_0^{\pi/2} (1+\cos\theta)^2\,d\theta$$

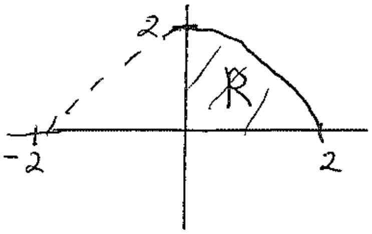
$$A = \frac{1}{2}\int_0^{\frac{\pi}{2}} [1 + 2\cos\theta + \cos^2\theta]\,d\theta$$ use formula #27

$$A = \frac{1}{2}\left[\theta + 2\sin\theta + \frac{1}{2}\theta + \frac{1}{4}\sin^2 2\theta\right]\Big|_0^{\pi/2}$$

$$A = \frac{1}{2}\left[\frac{\pi}{2} + 2 + \frac{\pi}{4} + 0\right] = \frac{3\pi}{8} + 1$$

5. (17 points) Evaluate the iterated integral by first converting it to polar coordinates.
$\int_0^2 \int_0^{\sqrt{4-x^2}} (x^2+y^2)\,dy\,dx$

$y=0$ to $y=\sqrt{4-x^2}$, $x=0$ to $x=2$



Convert $x^2+y^2=r^2$, ⊙ radius 2 is $r=2$

$$\int_0^{\frac{\pi}{2}}\int_0^2 r^2\, r\,dr\,d\theta = \int_0^{\frac{\pi}{2}} \frac{r^4}{4}\Big|_0^2 d\theta =$$

$$= 4\int_0^{\frac{\pi}{2}} d\theta = 4\cdot\frac{\pi}{2} = 2\pi$$

6. (17 points) The length, width and height of a box are measured with errors of at most 3%, 4% and 5%, respectively. Use differentials to approximate the maximum percentage error in the calculated volume.

Vol of box $V =$ length × width × height

$V = lwh$

differentials $dV = wh\,dl + lh\,dw + lw\,dh$

÷ by $V = lwh$, $\frac{dV}{V} = \frac{wh\,dl}{lwh} + \frac{lh\,dw}{lwh} + \frac{lw\,dh}{lwh}$

$$\frac{dV}{V} = \frac{dl}{l} + \frac{dw}{w} + \frac{dh}{h}$$

So $\frac{\Delta V}{V} \approx \frac{\Delta l}{l} + \frac{\Delta w}{w} + \frac{\Delta h}{h} = .03 + .04 + .05 = .12$

Max % error 12%