Calculus III	Test III	April 3, 2017
Dr. Platte
Name:________________________

1. (17 points) Find equation of the plane containing the points (1,1,1), (2,-1,3) and (2,1,0).

$\overrightarrow{P_1P_2} = \langle 1, -2, 2 \rangle$, $\overrightarrow{P_1P_3} = \langle 1, 0, -1 \rangle$ $\quad P_1 \quad P_2 \quad P_3$

$\overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3}$ is normal to the plane.

$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & -2 & 2 \\ 1 & 0 & -1 \end{vmatrix} = 2i + 3j + 2k$$

$\therefore$ Eq. of plane $2(x-1) + 3(y-1) + 2(z-1) = 0$

or $2x + 3y + 2z = 7$

2. (17 points) Find the gradient of $F(x,y,z) = xz + y^2z - 5$ at the point (1,2,1). Then find the equation of the tangent plane to the level surface $F(xyz) = 0$ (or $xz + y^2z = 5$) at (1,2,1).

$F_x = z$, $F_y = 2yz$, $F_z = x + y^2$ at $(1,2,1)$ $F_x = 1$, $F_y = 4$, $F_z = 5$

$\nabla F = i + 4j + 5k$. The gradient is normal to the level surface so also normal to tangent plane.

plane $(x-1) + 4(y-2) + 5(z-1) = 0$ or $x + 4y + 5z = 14$

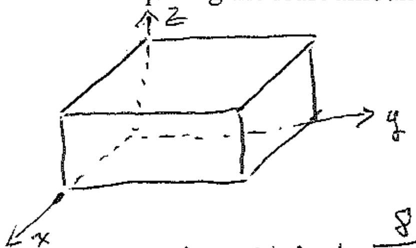
3. (17 points) Find the directional derivative of $f(x, y) = x^2y - xy^3$ at $(2, 1)$ in the direction of $1\mathbf{i} - 1\mathbf{j}$.

$f_x = 2xy - y^3$, $f_y = x^2 - 3xy^2$, at $(2,1)$ $f_x = 3$, $f_y = -2$

$\vec{\nabla} f = 3\hat{i} - 2\hat{j}$, unit vector in direction is $\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j} = \vec{u}$

$D_{\vec{u}} f = \vec{\nabla} f \cdot \vec{u} = \frac{3}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{5}{\sqrt{2}} = \frac{\sqrt{2}\,5}{2}$

4. (17 points) Find the dimension of a rectangular box, <u>open at the top</u>, having a volume of 4 ft^3, and requiring the least amount of material (surface area) for its construction.



Vol $= 4 = xyz$, Surface area $= SA$

$SA = xy + 2xz + 2yz$, substitute $\frac{4}{xy} = z$

$SA = xy + \frac{8}{y} + \frac{8}{x}$

$\frac{\partial SA}{\partial x} = y - \frac{8}{x^2} = 0$, $\frac{\partial SA}{\partial y} = x - \frac{8}{y^2} = 0$

$y = \frac{8}{x^2}$ substitute $x - \frac{8}{\left(\frac{8}{x^2}\right)^2} = 0$ or $x - \frac{x^4}{8} = 0$

$x(8 - x^3) = 0$ either $x = 0$ no box or $x = 2$.

If $x = 2$, $y = 2$ and $z = 1$. This is dimension of box with minamal surface area.

5. (15 points) Find equation of the line perpendicular to the plane $2x - y + 2z = 4$ and through the point (3,2,0).

normal vector to plane is $\vec{n} = 2i - j + 2k$. Vector is also // to line.

Line: $x = 3 + 2t$, $y = 2 - t$, $z = 2t$

6. (17 points) Locate all relative maxima, relative minima, and saddle points, if any, for

$f(x,y) = \frac{1}{3}x^3 - xy + \frac{1}{2}y^2 - 5y - x$. Be careful, make sure you get all critical points

Second Partial Test (x_0, y_0) is a critical point. $D = f_{xx}f_{yy} - {f_{xy}}^2$

i. $D > 0$, $f_{xx} > 0$ then rel. min. at (x_0, y_0)
ii. $D > 0$, $f_{xx} < 0$ then rel. max. at (x_0, y_0)
iii. $D < 0$, saddle point at (x_0, y_0)
iv. $D = 0$, test inconclusive

Find critical pts. $f_x = x^2 - y - 1$, $f_y = -x + y - 5$

Set to zero and solve for x and y.

$y = x^2 - 1$, $-x + y - 5 = 0$, so $-x + x^2 - 1 - 5 = 0$

$x^2 - x - 6 = 0$, factor $(x-3)(x+2) = 0$. Either $x = 3, -2$

critical pts $(3, 8)$ and $(-2, 3)$

$f_{xx} = 2x$, $f_{yy} = 1$, $f_{xy} = -1$, $D = 2x - 1$

at $(3,8)$, $D = 5 > 0$, $f_{xx} = 6 > 0$ $\therefore$ rel. min.

at $(-2,3)$, $D = -5 < 0$ $\therefore$ saddle pt.