Calculus III   Test III
Dr. Platte   April 8, 2014   Name:______________________

1. (14 points) Find the angle (to the nearest tenth of a degree) between the vectors **i** + 2**j** + 2**k** and 3**i** + 2**j** - 6**k**.

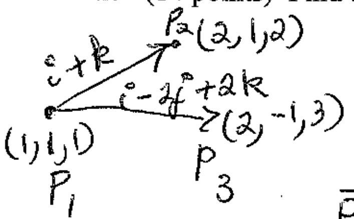
$\vec{u}\cdot\vec{v} = \|\vec{u}\|\,\|\vec{v}\|\cos\theta$

$\vec{u}\cdot\vec{v} = (i+2j+2k)\cdot(3i+2j-6k) = 3+4-12 = -5$

$\|i+2j+2k\| = \sqrt{9} = 3$, $\|3i+2j-6k\| = \sqrt{9+4+36} = \sqrt{49} = 7$

$\frac{-5}{3\cdot 7} = \cos\theta$, $\theta = 103.77^\circ = 103.8^\circ$

2. (14 points) Find the equation of the plane containing the points (1, 1, 1), (2, 1, 2) and (2, -1, 3).



$\vec{P_1P_2} = i + k$, $\vec{P_1P_3} = i - 2j + 2k$

Cross product normal to desired plane

$\vec{P_1P_2}\times\vec{P_1P_3} = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 1 & -2 & 2 \end{vmatrix} = +2i - j - 2k$ or $2i + j + 2k$

plane $-2(x-1)+1(y-1)+2(z-1) = 0$

$-2x + y + 2z = 1$

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or

$2x - y - 2z = -1$

3. (14 points) Find a unit vector normal to plane $3x + 2y - \sqrt{3}\,z = 4$.

a normal vector is $\vec{n} = \langle 3, 2, -\sqrt{3}\rangle$

It's norm is $\|\vec{n}\| = \sqrt{9+4+3} = 4$

So unit normal $\frac{\vec{n}}{\|\vec{n}\|} = \langle \frac{3}{4}, \frac{2}{4}, -\frac{\sqrt{3}}{4} \rangle$

4. (14 points) Find the equation of the tangent plane to the surface $f(x,y) = z = 25 - x^2 - y^2$ at (4, -2).

Two forms for normal and plane!

If $z = f(x,y)$ use $\vec{n} = f_x \hat{i} + f_y \hat{j} - k$

If $F(x,y,z) = $ constant use $\vec{n} = F_x \hat{i} + F_y \hat{j} + F_z \hat{i}$

I'll change problem to use second form! ie $F(x,y,z) = x^2 + y^2 + z = 25$

$\vec{n} = F$, $F_x = 2x$, $F_y = 2y$, $F_z = 1$ at $(4,-2)$

$F_x = 8$, $F_y = -4$, $F_z = 1$ and $z = 25 - 16 - 4 = 5$. Point on surface $(4,-2,5)$

Eq. of tangent plane $8(x-4) - 4(y+2) + 1(z-5) = 0$

$8x - 4y + z = 45$

or $-8x + 4y - z = -45$

or $-8x + 4y - z + 45 = 0$

5. (15 points) Locate and identify all relative maxima, relative minima and saddle points for the function below. Prove the claimed points are relative extreme points by using the second partial derivative test.

$f(x,y) = x^2 - xy + y^2 + 4y - 2x$

$f_x = 2x - y - 2$ $\left\{ \begin{array}{l} \text{Set to zero and} \\ \text{solve for } x \text{ and } y \end{array} \right\}$ $\left. \begin{array}{l} 2x - y - 2 = 0 \\ -2x + 4y + 8 = 0 \end{array} \right\}$ add $3y = 6$, $y = 2$

$f_y = -x + 2y + 4$

If $y = -2$ so $x = 0$ critical point $(0, -2)$

$f_{xx} = 2$, $f_y = 2$, $f_{xy} = -1$ so $D = f_{xx} f_{yy} - f_{xy}^2 = 2 \cdot 2 - 1 = 3$

$f_{xx} > 0$, $D > 0$ so relative min at $(0,-2)$.

Note: at $(0,-2)$, $f(0,-2) = -4$

6. (14 points) Find the gradient of f(x,y) at P, and then use the gradient to find the directional derivative at P in the direction of **u**. $z = f(x,y) = x^2y^2 + y - xy$, $P(2,-1)$, $u = -\frac{\sqrt{3}}{2}i + \frac{1}{2}j$

$f_x = 2xy^2 - y$ at $(2,-1)$ $f_x = 5$

$f_y = 2x^2y + 1 - x$ at $(2,-1)$ $f_y = -9$

So $\vec{\nabla} f = 5\hat{i} + \cancel{8} - 9\hat{j}$

$D_u = \vec{\nabla} f \cdot \vec{u} = -\frac{5\sqrt{3}}{2} - \frac{9}{2} = -\frac{9+5\sqrt{3}}{2}$

7. (15 points) Use Lagrange multipliers to find the maximum and minimum values of f(x,y) subject to the constraint. $f(x,y) = 4x^3 + 3y^2$; Constraint: $2x^2 + y^2 = 4$ Hint: Find value of lambda λ first.

$\vec{\nabla} f = 12x^2\hat{i} + 6y\hat{j}$, $\vec{\nabla} g = 4x\hat{i} + 2y\hat{j}$

$\vec{\nabla} f = \lambda \vec{\nabla} g$ so $12x^2 = 4x\lambda$ and $6y = 2y\lambda$

These equations become $4x(3x - \lambda) = 0$, $2y(3 - \lambda) = 0$

From 2nd eq. either $y = 0$ or $\lambda = 3$.
From 1st eq either $x = 0$ or $3x = \lambda$.
$\lambda = 3$ means $x = 1$. Substitute $x = 0, x = 1, y = 0$ into constraint eq. and get 6 points

point	f(x,y)	
$(0,2)$	12	rel max
$(0,-2)$	12	rel max
$(\sqrt{2},0)$	$4 \cdot 2^{3/2} = 11.31$	
$(-\sqrt{2},0)$	$-4 \cdot 2^{3/2} = -11.31$	rel min
$(1,\sqrt{2})$	10	
$(1,-\sqrt{2})$	10	