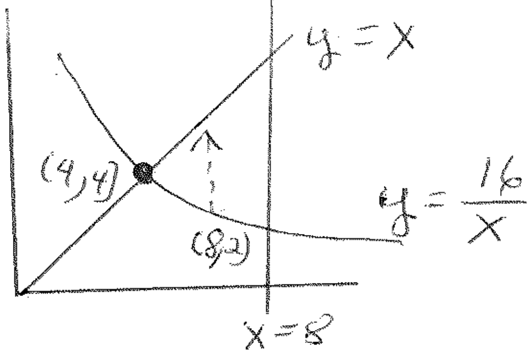
15a)

Page 1016



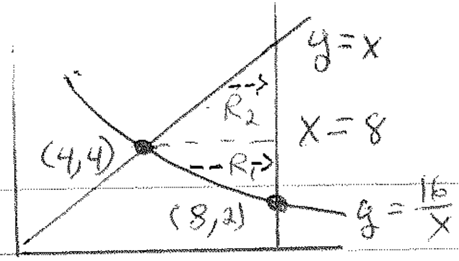
As type I region $V = \int_4^8 \int_{16/x}^{x} x^2 \, dy \, dx =$

$\int_4^8 x^2 y \Big|_{16/x}^{x} dx = \int_4^8 (x^3 - 16x) \, dx =$

$\left(\frac{x^4}{4} - 8x^2\right)\Big|_4^8 = 2 \cdot 8^3 - 8^3 - 4^3 + 8 \cdot 4^2 = 8^3 + 4^3 = 512 + 64$

576

15b)



As type II double integrals there are 2 regions, $R_1 \cup R_2$

Vol over R_1

$$\int_2^4 \int_{\frac{16}{y}}^8 x^2\,dx\,dy =$$

$$\int_2^4 \frac{x^3}{3}\Big|_{\frac{16}{y}}^8 dy = \int_2^4 \left(\frac{8^3}{3} - \frac{(2\cdot 8)^3}{3\cdot y^3}\right)dy = \frac{8^3}{3}\int_2^4 \left(1 - \frac{8}{y^3}\right)dy$$

$$= \frac{8^3}{3}\left(y + \frac{4}{y^2}\right)\Big|_2^4 = \frac{8^3}{3}\left[4 + \frac{1}{4} - 2 - 1\right] = \frac{8^3}{3}\cdot\frac{5}{4} = \frac{640}{3}$$

Vol over R_2

$$\int_4^8 \int_y^8 x^2\,dx\,dy =$$

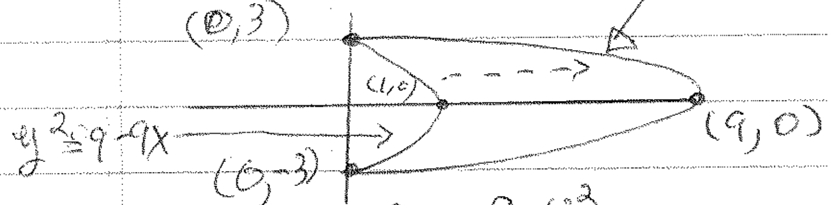
$$\int_4^8 \frac{x^3}{3}\Big|_y^8 dy = \frac{1}{3}\int_4^8 (8^3 - y^3)\,dy =$$

$$\frac{1}{3}\left(8^3 y - \frac{y^4}{4}\right)\Big|_4^8 = \frac{1}{3}\left[8^4 - \frac{8^4}{4} - 8^3\cdot 4 + 4^3\right]$$

$$= \frac{1}{3}\left[4096 - 1024 - 2048 + 64\right] = \frac{1088}{3}$$

Vol = Vol over R_1 + Vol over R_2 = $\frac{640}{3} + \frac{1088}{3} = 576$

#31 area between $y^2 = 9 - x$ and $y^2 = 9 - 9x$



Fix y vary x

$$V = \int_{-3}^{3} \int_{\frac{9-y^2}{9}}^{9-y^2} 1 \, dx\,dy =$$

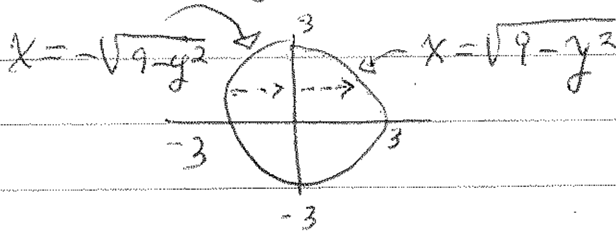
$$\int_{-3}^{3} \left[9 - y^2 - 1 + \frac{y^2}{9}\right] dy = \int_{-3}^{3} \left(8 - \frac{8}{9}y^2\right) dy$$

$$\left(8y - \frac{8y^3}{27}\right)\Big|_{-3}^{3} = \left(24 - 8 + 24 - 8\right) = 32$$

Note: Difficult to do as type I

ie $\iint_R 1 \, dy\,dx$

39 Region R, inside $x^2+y^2=9$, surface $z=3-x$.



$$V=\int_{-3}^{3}\int_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}(3-x)\,dx\,dy$$

$$=\int_{-3}^{3}\left(3x-\frac{x^2}{2}\right)\Bigg|_{-\sqrt{9-y^2}}^{\sqrt{9-y^2}}dy=$$

$$\int_{-3}^{3}\left[3\sqrt{9-y^2}-\frac{9-y^2}{2}+3\sqrt{9-y^2}+\frac{9-y^2}{2}\right]dy$$

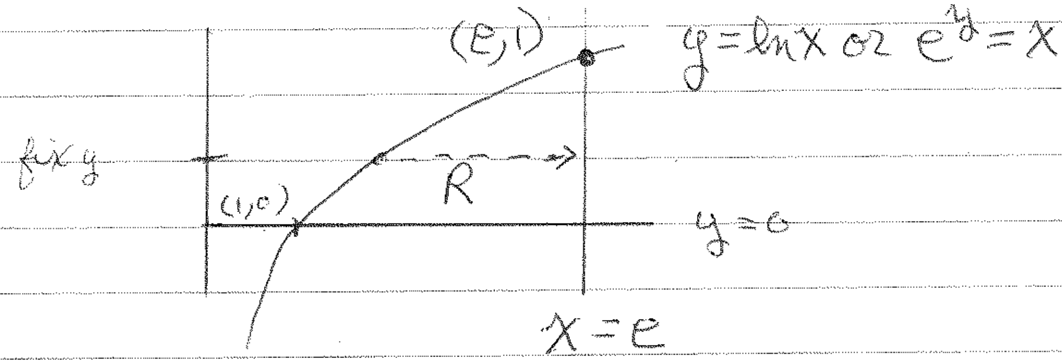
$$=\int_{-3}^{3}6\sqrt{9-y^2}\,dy=6\left(\frac{y}{2}\sqrt{9-y^2}+\frac{9}{2}\sin^{-1}\frac{y}{3}\right)\Bigg|_{-3}^{3}=$$

$$=6\left[0+\frac{9}{2}\sin^{-1}(1)-0-\frac{9}{2}\sin^{-1}(-1)\right]=$$

$$6\left[\frac{9}{2}\cdot\frac{\pi}{2}-\frac{9}{2}\left(-\frac{\pi}{2}\right)\right]=6\cdot\frac{9}{2}\cdot\frac{\pi}{2}\cdot 2=27\pi$$

50. $\int_1^e \int_0^{\ln x} f(x,y)\,dy\,dx$

Must find region R: $y=0$ to $y=\ln x$, $x=1$ to $x=e$



$$\int_0^1 \int_{e^y}^{e} f(x,y)\,dx\,dy$$

fix y