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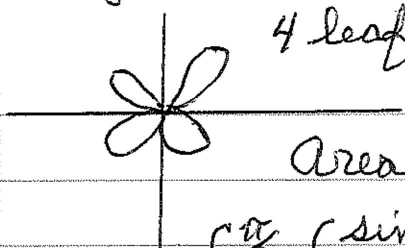
#8 4 leaf rose, $r = \sin 2\theta$, Find area.



area of 4 leafs = 4 × area of 1st quad leaf.

$$A = 4\int_0^{\pi/2}\int_0^{\sin 2\theta} 1 \; r\,dr\,d\theta =$$

$$A = 4\int_0^{\pi/2} \tfrac{1}{2}\sin^2 2\theta \, d\theta = 2\int_0^{\pi/2} \sin^2 2\theta \, d\theta$$

Must do this! Let $u = 2\theta$, $\frac{1}{2}du = d\theta$. If $\theta = 0, u = 0$. If $\theta = \frac{\pi}{2}, u = \pi$

$$A = \int_0^{\pi} \sin^2 u \, du \qquad \text{use formula 26}$$

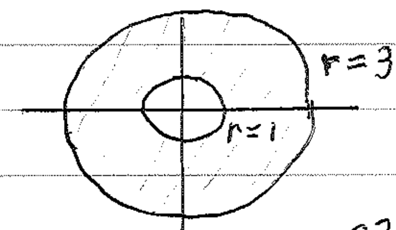
$$A = \left[\tfrac{1}{2}u - \tfrac{1}{4}\sin 2u\right]\Big|_0^{\pi} = \pi/2$$

#13 sphere $x^2+y^2+z^2=9$, upper half $z = \sqrt{9-x^2-y^2}$

cylinder $x^2+y^2=1$

Convert to polar $z = \sqrt{9-r^2}$ and $r = 1$

Region



Volume above xy plane $\int_0^{2\pi}\int_1^{3} \sqrt{9-r^2}\; r\,dr\,d\theta$

Double this for total volume!

$$V = 2\int_0^{2\pi}\int_1^{3} \sqrt{9-r^2}\; r\,dr\,d\theta$$

#17 Let $u = 9 - r^2$, $du = -2r\,dr$, $-\frac{1}{2}du = r\,dr$

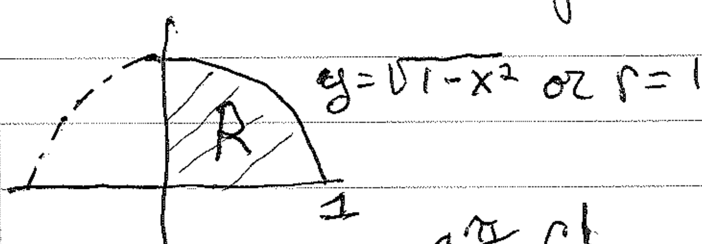
$$V = -1\int_0^{2\pi}\int u^{\frac{1}{2}}\,du\,d\theta = -1\int_0^{2\pi} \frac{2}{3}u^{\frac{3}{2}}\Big|\,d\theta =$$

$$V = -1\cdot\frac{2}{3}\int_0^{2\pi}(9-r^2)^{\frac{3}{2}}\Big|_1^3\,d\theta = -\frac{2}{3}\int_0^{2\pi} -8^{\frac{3}{2}}\,d\theta$$

$$= \frac{2}{3}\cdot 16\sqrt{2}\cdot 2\pi = \frac{64\sqrt{2}\,\pi}{3}$$

#27 $$\int_0^1\int_0^{\sqrt{1-x^2}}(x^2+y^2)\,dy\,dx$$

First determine region! $y=0$ to $y=\sqrt{1-x^2}$



Polar Form $\displaystyle\int_0^{\frac{\pi}{2}}\int_0^1 r^2\,r\,dr\,d\theta =$

$$\int_0^{\frac{\pi}{2}}\frac{r^4}{4}\Big|_0^1\,d\theta = \frac{1}{4}\int_0^{\frac{\pi}{2}}d\theta = \frac{\pi}{8}$$