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9. $\iiint_G xy \sin yz \, dV$, $0 \le x \le \pi$, $0 \le y \le 1$, $0 \le z \le \frac{\pi}{6}$

Integration w.r.t. y should be last so either $dx\,dz\,dy$ or $dz\,dx\,dy$.

$$\int_0^1 \int_0^{\frac{\pi}{6}} \int_0^{\pi} xy \sin zy \, dx\,dz\,dy = \int_0^1 \int_0^{\frac{\pi}{6}} \frac{x^2}{2} y \sin zy \Big|_0^{\pi} dz\,dy$$

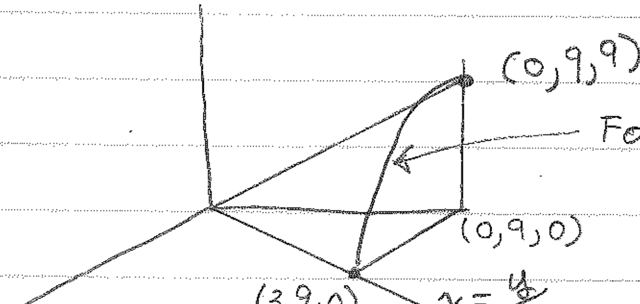
$$\frac{\pi^2}{2} \int_0^1 \int_0^{\frac{\pi}{6}} y \sin zy \, dz\,dy = \frac{\pi^2}{2} \int_0^1 -\cos zy \Big|_0^{\frac{\pi}{6}} dy$$

$$= \frac{\pi^2}{2} \int_0^1 \left(-\cos \frac{\pi}{6} y + 1\right) dy = \frac{\pi^2}{2}\left(-\frac{6}{\pi} \sin \frac{\pi}{6} y + y\right)\Big|_0^1$$

$$= \frac{\pi^2}{2}\left(-\frac{6}{\pi} \cdot \frac{1}{2} + 1\right) = \frac{\pi^2}{2}\left(1 - \frac{3}{\pi}\right) = \frac{\pi}{2}(\pi - 3)$$

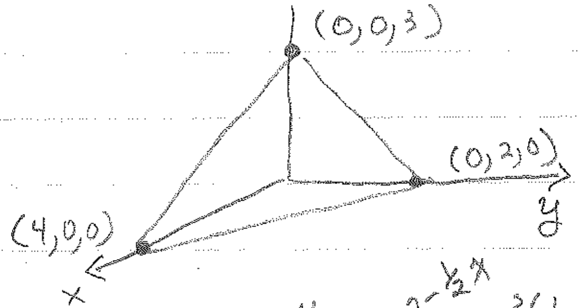
25b) $\int_0^9 \int_0^{y/3} \int_0^{\sqrt{y^2 - 9x^2}} dz\,dx\,dy$

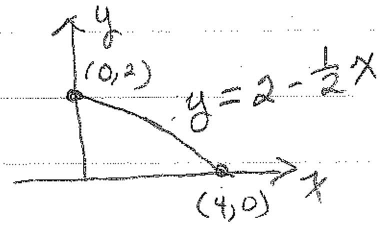
z varies from $z = 0$ to $z = \sqrt{y^2 - 9x^2}$.
So $z^2 = y^2 - 9x^2$
x varies from $x = 0$ to $x = \frac{y}{3}$
z varies from $z = 0$ to $z = 9$



For y = 9 this is part of an ellipse $\frac{z^2}{81} + \frac{x^2}{9} = 1$

15. $3x+6y+4z=12$, $z = 3-\frac{3}{4}x-\frac{3}{2}y = 3(1-\frac{1}{4}x-\frac{1}{2}y)$





Vol is $\frac{1}{6}$ vol of box $4\times 2\times 3$. ie 4

$$\int_0^4 \int_0^{2-\frac{1}{2}x} \int_0^{3(1-\frac{1}{4}x-\frac{1}{2}y)} 1 \, dz\, dy\, dx =$$

$$\int_0^4 \int_0^{2-\frac{1}{2}x} 3(1-\tfrac{1}{4}x-\tfrac{1}{2}y)\, dy\, dx =$$

$$* \quad 3\int_0^4 \left[(1-\tfrac{1}{4}x)y - \tfrac{1}{4}y^2\right]\Big|_0^{2-\frac{1}{2}x} dx =$$

$$3\int_0^4 \left[(1-\tfrac{1}{4}x)2(1-\tfrac{1}{4}x) - \tfrac{1}{4}\cdot 4(1-\tfrac{1}{4}x)^2\right] dx =$$

$$3\int_0^4 (1-\tfrac{1}{4}x)^2\, dx = 3\,\frac{(1-\frac{1}{4}x)^3}{3}\left(-\frac{4}{1}\right)\Bigg|_0^4 = 4$$

OR

$$* \quad 3\int_0^4 \left[(1-\tfrac{1}{4}x)(2-\tfrac{1}{2}x) - \tfrac{1}{4}(2-\tfrac{1}{2}x)^2\right] dx =$$

$$3\int_0^4 \left[(2-x+\tfrac{1}{8}x^2) - \tfrac{1}{4}(4-2x+\tfrac{1}{4}x^2)\right] dx =$$

$$3\int_0^4 \left[2-x+\tfrac{1}{8}x^2-1+\tfrac{1}{2}x-\tfrac{1}{16}x^2\right] dx =$$

$$3\int_0^4 \left[1-\tfrac{1}{2}x+\tfrac{1}{16}x^2\right] dx = 3\left[x-\tfrac{1}{4}x^2+\tfrac{1}{16}\,\tfrac{x^3}{3}\right]\Big|_0^4$$

$$= 3\left[4-\tfrac{1}{4}\cdot 4^2+\tfrac{1}{4^2}\,\tfrac{4^3}{3}\right] = 3\left[4-4+\tfrac{4}{3}\right] = 4$$