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$$\int_{(-1,2)}^{(0,1)} (3x-y+1)dx - (x+4y+2)dy$$

Independent of path if $\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$

$f = 3x - y + 1$, $f_y = -1$
$g = -x - 4y - 2$, $g_x = -1$

$\therefore$ Independent of path.

So we are able to find $\phi(x,y)$ s.t.

$$\vec{\nabla}\phi = \vec{F(x,y)} = (3x-y+1)\hat{i} - (x+4y+2)\hat{j}$$

$$\frac{\partial\phi}{\partial x} = 3x - y + 1 \implies \phi = \tfrac{3}{2}x^2 - xy + x + h_1(y)$$

$$\frac{\partial\phi}{\partial y} = -x - 4y - 2 \implies \phi = -xy - 2y^2 - 2y + h_2(x)$$

So $h_1(y) = -2y^2 - 2y$ and $h_2(x) = \frac{3}{2}x^2 + x$

and $\phi(x,y) = -xy + \frac{3}{2}x^2 + x - 2y^2 - 2y$

By Fundamental Theorem for line integrals

$$\int_{(-1,2)}^{(0,1)} (3x-y+1)dx - (x+4y+2)dy =$$

$$\left(-0\cdot1 + \tfrac{3}{2}0^2 + 0 - 2 - 2\right) - \left(-(-1)\cdot2 + \tfrac{3}{2}\cdot1 - 1 - 8 - 4\right)$$

$$-4 - \left(2 + \tfrac{3}{2} - 13\right) = -4 - \left(\tfrac{7}{2} - \tfrac{26}{2}\right) = -4 + \tfrac{19}{2} = \tfrac{11}{2}$$

#16 $\overrightarrow{F(x,y)} = 2xy^3\hat{i} + 3x^2y^2\hat{j}$, $P(-3,0)$, $Q(4,1)$

So $f(x,y) = 2xy^3$, $f_y = 6xy^2$

$g(x,y) = 3x^2y^2$, $g_x = 6xy^2$

$\therefore$ conservative and independent of path.

So $\phi(x,y)$ exists s.t. $\vec{\nabla\phi} = \vec{F}$.

$\frac{\partial\phi}{\partial x} = 2xy^3 \Longrightarrow \phi = x^2y^3 + h_1(y)$

$\frac{\partial\phi}{\partial x} = 3x^2y^2 \Longrightarrow \phi = x^2y^3 + h_2(x)$

The $h_1(y)$ and $h_2(x)$ are not necessary here.

$\phi = x^2y^3$

Work is $\int_P^Q \vec{F}\cdot\vec{dr}$. The Fund. Thm for line integrals apply.

So $W = \int_{(-3,0)}^{(4,1)} \vec{F}\cdot\vec{dr} = x^2y^3 \Big|_{(-3,0)}^{(4,1)} = 16 - 0 = 16$