#11 $\oint \tan^{-1} y\, dx - \frac{y^2 x}{1+y^2} dy$, C is a square vertices $(0,0)$, $(1,0)$, $(1,1)$, $(0,1)$

$f = \tan^{-1} y \qquad g = -\frac{y^2 x}{1+y^2}$

For Green's Thm need $\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}$

$\frac{\partial g}{\partial x} = -\frac{y^2}{1+y^2}$, $\frac{\partial f}{\partial y} = \frac{1}{y^2+1}$

So $\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} = -\frac{y^2}{1+y^2} - \frac{1}{1+y^2} = -\frac{1+y^2}{1+y^2} = -1$

$\therefore \oint \tan^{-1} y\, dx - \frac{y^2 x}{1-y^2} dy = \iint_{\text{square}} -1\, dA$

$= \int_0^1 \int_0^1 -1\, dx\, dy = -1$

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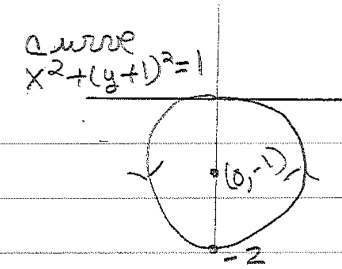

$\oint_c y\tan^2 x\,dx + \tan x\,dy$

$f(x,y) = y\tan^2 x$, $g(x,y) = \tan x$

$\frac{\partial f}{\partial y} = \tan^2 x$, $\frac{\partial g}{\partial x} = \sec^2 x$

So Green's Thm $\iint_R (\sec^2 x - \tan^2 x)\,dA$

using trig id $\sec^2 x - \tan^2 x = 1$

So integral becomes $\iint_R 1\,dA$. This is area of the ⊙

ie $\iint_R 1\,dA = \pi$ answer.

If you want to do calculus to find $\iint_R 1\,dA$,

It would be $\int_{-2}^{0}\int_{-\sqrt{1-(y+1)^2}}^{\sqrt{1-(y+1)^2}} 1\,dx\,dy$. This would be

difficult to do. So convert to polar

$x^2+(y+1)^2=1$, or $x^2+y^2+2y+1=1$, or $r^2+2r\sin\theta=0$

or $r=-2\sin\theta$.

$\int_0^{\pi}\int_0^{-2\sin\theta} 1\,r\,dr\,d\theta = \int_0^{\pi}\frac{2^2\sin^2\theta}{2}\,d\theta = 2\int_0^{\pi}\sin^2\theta\,d\theta$

$= 2\left(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right)\Big|_0^{\pi} = \pi$