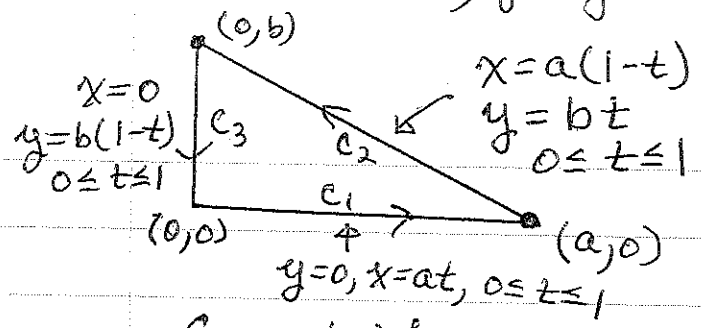
Problem 22, page 1128



Green's Theorem

$$\iint_R dA = \frac{1}{2}\oint (-y\,dx + x\,dy)$$

You were asked to evaluate the left right hand side and show it equals left hand side. Left hand side is clearly $\frac{1}{2}ab$.

The closed curve is the piecewise path C_1 to C_2 to C_3.

For path C_1 can easily show $\frac{1}{2}\int_{C_1} -y\,dx + x\,dy = 0$.

For path C_3 can easily show $\frac{1}{2}\int_{C_3} -y\,dx + x\,dy = 0$.

For path C_2: $\frac{1}{2}\int_{C_2} -y\,dx + x\,dy =$

$$\frac{1}{2}\int_0^1 -bt(-a\,dt) + a(1-t)b\,dt =$$

$$\frac{1}{2}\int_0^1 [abt\,dt + ab\,dt - abt\,dt] = \frac{1}{2}\int_0^1 ab\,dt =$$

$$\frac{1}{2}ab.$$