2. $\overrightarrow{F(x,y,z)} = x\hat{i} + y\hat{j} + z\vec{k}$, $z = \sqrt{a^2 - x^2 - y^2}$

As a line Integral: $x = a\cos t$, $y = a\sin t$, $z=0$ this is a parameterization of curve $x^2 + y^2 = a^2$.

$$\vec{F} = a\cos t\, \hat{i} + a\sin t\, \hat{j} + 0k$$

$$\overrightarrow{dr} = dx\,\hat{i} + dy\,\hat{j} + 0k$$

$$dr = -a\sin t\,dt\,\hat{i} + a\cos t\,dt\,\hat{j} + 0k$$

$$\vec{F} \cdot \overrightarrow{dr} = -a^2 \sin t \cos t\, dt + a^2 \cos t \sin t\, dt$$

$$\int_0^{2\pi} \vec{F} \cdot \overrightarrow{dr} = a^2 \int_0^{2\pi} \left(-\sin t \cos t\, dt + \cos t \cdot \sin t\, dt\right) = a^2 \Big|_0^{2\pi} 0 = 0$$

Using Stoke's Thm

$$\overrightarrow{\text{curl}\, F} = \begin{vmatrix} \hat{i} & \hat{j} & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = \vec{0} \quad \text{it is a vector!}$$

$$\text{normal } \vec{n} = \frac{\partial z}{\partial x}\hat{i} + \frac{\partial z}{\partial y}\hat{j} - k =$$

$$-\frac{x}{\sqrt{a^2 - x^2 - y^2}}\hat{i} - \frac{y}{\sqrt{a^2 - x^2 - y^2}}\hat{j} - k.$$

However this normal points "downward"!

Correct normal: $\frac{x}{\sqrt{a^2 - x^2 - y^2}}\hat{i} + \frac{y}{\sqrt{a^2 - x^2 - y^2}}\hat{j} + k$

$$\iint_A \overrightarrow{\text{curl}\, F} \cdot \vec{n}\, dA = \iint_A 0\, dA = 0$$