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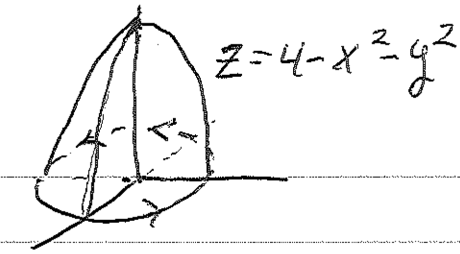


$\vec{F} = 3z\,i + 4x\,j + 2y\,k$

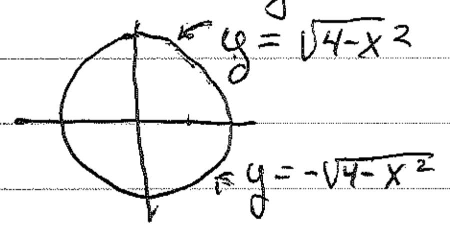
$$\vec{\nabla} \times \vec{F} = \overrightarrow{\text{curl}\,F} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3z & 4x & 2y \end{vmatrix} = 2i + 3j + 4k$$

Normal points up $\vec{n} = \langle -\sigma_x, -\sigma_y, 1 \rangle$
here $\sigma = 4 - x^2 - y^2$

$\vec{n} = \langle 2x, 2y, 1 \rangle$

$$\oint \vec{F} \cdot \vec{dr} = \iint_A \overrightarrow{\text{curl}\,F} \cdot \vec{n}\, ds = \iint_A (4x + 6y + 4)\, dA$$

In rectangular Coordinates



$$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (4x + 6y + 4)\, dy\, dx$$

In polar coordinates $x = r\cos\theta$, $y = r\sin\theta$

$$\int_0^{2\pi} \int_0^2 (4r\cos\theta + 6r\sin\theta + 4)\, r\, dr\, d\theta$$

$$\int_0^{2\pi} \left(\frac{4}{3} r^3 \cos\theta + 2r^3 \sin\theta + 2r^2 \right) \Big|_0^2 \, d\theta$$

$$8\int_0^{2\pi}\left(\tfrac{4}{3}\cos\theta + 2\sin\theta + 1\right)d\theta$$

$$= 8\left(-\tfrac{4}{3}\sin\theta + 2\cos\theta + \theta\right)\Big|_0^{2\pi}$$

$$= 8\left(0 + 0 + 2\pi\right) = 16\pi$$