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#23 $f = \frac{x+y}{xy}$, $f_x = \frac{xy \cdot 1 - (x+y)y}{(xy)^2} = -\frac{1}{x^2}$

$f_y = -\frac{1}{y^2}$, $f(-1,-2) = \frac{3}{2}$, $f_x(-1,-2) = -1$, $f_y(-1,-2) = -\frac{1}{4}$

$\Delta x = (-1.02 - (-1)) = -.02$, $\Delta y = (-2.04 - (-2)) = -.04$

Local linear approx $f(-1.02, -2.04) \approx -\frac{3}{2} - 1 \cdot (-.02) - \frac{1}{4}(-.04)$

$f(-1.02, 2.04) \approx -1.5 + .02 + .01 = 1.47$

#37 $f = xyz$, $f_x = yz$, $f_y = xz$, $f_z = xy$

at $(1,2,3)$, $f = 6$, $f_x = 6$, $f_y = 3$, $f_z = 2$

$\Delta x = 1.001 - 1 = .001$, $\Delta y = 2.002 - 2 = .002$, $\Delta z = 3.003 - 3 = .003$

Local linear approx

$f(1.001, 2.002, 3.003) \approx 6 + 6 \times .001 + 3 \times .002 + 2 \times .003$

$\approx 6 + .006 + .006 + .006 = 6.018$

b) Actual $f(1.001, 2.002, 3.003) = 6.018\,018\,006$

#54 $V = \frac{1}{3}\pi r^2 h$

$\frac{\partial V}{\partial r} = V_r = \frac{1}{3}\pi h \cdot 2r$ $\frac{\partial V}{\partial h} = V_h = \frac{1}{3}\pi r^2$

total differential $dV = V_r \cdot dr + V_h \cdot dh$

$dV = \frac{2}{3}\pi r h \, dr + \frac{1}{3}\pi r^2 dh = \frac{1}{3}\pi(2rh\,dr + r^2 dh)$

divide by V

$$\frac{dV}{V} = \frac{\frac{1}{3}\pi}{\frac{1}{3}\pi}\left(\frac{2rh\,dr}{r^2 h} + \frac{r^2 dh}{r^2 h}\right) = 2\frac{dr}{r} + \frac{dh}{h}$$

$\therefore$ Max. % error $\left|\frac{dV}{V}\right| \times 100\% = 2 \cdot 1\% + 4\% = 6\%$