Page 968

#11 $f = y^2 \ln x$, $P(1,4)$, $\vec{a} = -3\vec{i} + 3\vec{j}$

unit vector $\vec{u} = -\frac{1}{\sqrt{2}}\vec{i} + \frac{1}{\sqrt{2}}\vec{j}$

$\nabla f = \frac{y^2}{x}\vec{i} + 2y \ln x \vec{j}$, at $(1,4)$, $\nabla f = 16\vec{i}$

$D_{\vec{u}} f = \nabla f \cdot \vec{u} = -\frac{16}{\sqrt{2}}$ or $-8\sqrt{2}$

#25 $f = \sqrt{xy}\, e^y = x^{\frac{1}{2}} y^{\frac{1}{2}} e^y$ at $(1,1)$ in direction $-\vec{j}$.

$\nabla f = \frac{1}{2}\left(\frac{y}{x}\right)^{\frac{1}{2}} e^y \vec{i} + \left(\frac{1}{2}\left(\frac{x}{y}\right)^{\frac{1}{2}} + (xy)^{\frac{1}{2}}\right) e^y \vec{j}$ at $(1,1)$

$\nabla f = \frac{e}{2}\vec{i} + \frac{3}{2}e\vec{j}$

$D_{-\vec{j}} f = -\frac{3}{2}e$

#44 $f(x,y) = (x^2+y^2)^{\frac{1}{2}}$

$f_x = -\frac{1}{2}(x^2+y^2)^{-\frac{3}{2}} \cdot 2x = \frac{-x}{(x^2+y^2)^{\frac{3}{2}}}$

$f_y = \frac{-y}{(x^2+y^2)^{\frac{3}{2}}}$ at $(3,4)$

$f_x = -\frac{3}{125}$, $f_y = -\frac{4}{125}$, $\nabla f = -\frac{1}{125}(3\vec{i} + 4\vec{j})$

#57. $f = x^3z^2 + y^3z + z - 1$ at $(1,1,-1)$

$f_x = 3x^2z^2$, $f_y = 3y^2z$, $f_z = 2x^3z + y^3 + 1$ at $(1,1,-1)$

$f_x = 3$, $f_y = -3$, $f_z = 0$

$\nabla f = 3\vec{i} - 3\vec{j}$, $\|\nabla f\| = 3\sqrt{2}$ which is rate of change

Direction of most rapid inc $\frac{\nabla f}{\|\nabla f\|} = \frac{1}{\sqrt{2}}(\vec{i} - \vec{j})$