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#4 $x^2y - 4z^2 = -7$, $P(-3, 1, -2)$

$F(x,y,z) = x^2y - 4z^2$, $\nabla F = 2xy\,i + x^2 j - 8z\,k$

at $(-3,1,-2)$ $\nabla F = -6i + 9j + 16k$

Eq. of tangent plane $-6(x+3) + 9(y-1) + 16(z+2) = 0$

or $6x + 9y + 16z + 15 = 0$

Parametric equations of normal line

$x = -3 - 6t$, $y = 1 + 9t$, $z = -2 + 16t$

#12. $z = x^{\frac{1}{2}} + y^{\frac{1}{2}}$, $P(4, 9, 5)$

Note: of form $z = f(x,y)$ so use $\vec{n} = f_x i + f_y j - k$

$n = \frac{1}{2}x^{-\frac{1}{2}} i + \frac{1}{2}y^{-\frac{1}{2}} j - k$, at $(4,9,5)$, $\vec{n} = \frac{1}{4}i + \frac{1}{6}j - k$

Eq. of tangent plane $\frac{1}{4}(x-4) + \frac{1}{6}(y-9) - (z-5) = 0$

or $\frac{1}{4}x + \frac{1}{6}y - z + \frac{5}{2} = 0$

or $3x + 2y - 12z + 30 = 0$

Parametric equations of normal line

$x = 4 + \frac{1}{4}t$, $y = 9 + \frac{1}{6}t$, $z = 5 - t$

#25 sphere $x^2 + y^2 + z^2 = 1$, $\vec{n} = 2x_0 i + 2y_0 j + 2z_0 k$ or

$\vec{n} = x_0 i + y_0 j + z_0 k$ where (x_0, y_0, z_0) on sphere.

Eqs of normal line: $x = x_0 + x_0 t$, $y = y_0 + y_0 t$, $z = z_0 + z_0 t$.

For any t value you get a point on the line. Letting $t = -1$ we see $(0,0,0)$ is on the line.