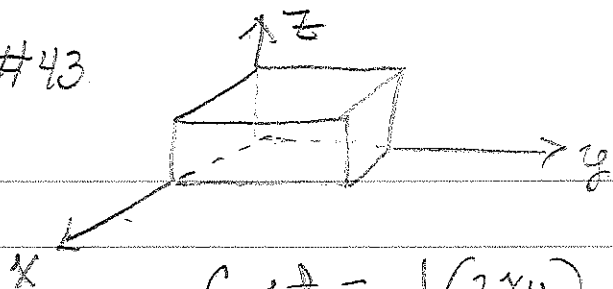


#43



$$\text{Vol} = 16 = xyz$$

$$\text{Cost} = .1(2xy) + .05(2xz) + .05(2yz), \text{ minimize!}$$

For the calculus simplify the above! Divide by .1!

$$C = \frac{\text{Cost}}{.1} = 2xy + xz + yz, \text{ minimize this!}$$

Subst.  $z = \frac{16}{xy}$  into above eq.

$$C = 2xy + \frac{16}{y} + \frac{16}{x}, \text{ obtain critical pt. (s)}$$

$$C_x = 2y - \frac{16}{x^2}, \quad C_y = 2x - \frac{16}{y^2}, \text{ set to 0 and solve}$$

$$\frac{y}{4} = \frac{8}{x^2}, \quad x = \frac{8}{y}, \text{ so } x = \frac{8}{\frac{8}{x^2}}, \text{ or } x = \frac{1}{8}x^4 \text{ or } 8x = x^4$$

Solving  $x(x^3 - 8) = 0$ , so  $x = 0$  or  $x = 2$ . But  $x$  can not be 0.

If  $x = 2$  then  $y = 2$  and  $z = 4$ .

From 2nd partial derivative test

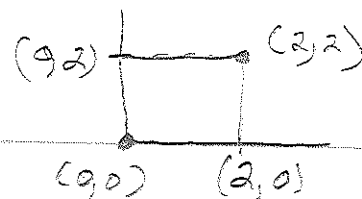
$$C_{xx} = 4 = C_{yy}, \quad C_{xy} = 2 \text{ so } D = 16 - 4 > 0$$

∴ local min at (2, 2)

I say there is no boundary! i.e.  $0 < x < 16$ ,  $0 < y < 16$ .  
Thus local min is actually

Abs. Min! Cost = \$2.40 at (2, 2, 4)

$f(x,y) = x^2 - 3y^2 - 2x + 6y$  on



1. Interior critical points

$f_x = 2x - 2 = 0 \implies x = 1$

$f_y = -6y + 6 = 0 \implies y = 1$

$(1,1)$  is only interior critical point

2. Boundary critical points

a)  $x = 0, 0 \leq y \leq 2$

function becomes  $f(0,y) = -3y^2 + 6y$ , one var. funct.

$\frac{df}{dy} = -6y + 6 = 0 \implies y = 1$ , so critical pt at  $(0,1)$ .

End pts also critical. Here  $(0,0)$  and  $(0,2)$ .

b)  $x = 2, 0 \leq y \leq 2$ . Work similar to above!  
critical pts  $(2,0), (2,1), (2,2)$

c)  $y = 0, 0 \leq x \leq 2$

function becomes  $f(x,0) = x^2 - 2x$ , one var. funct

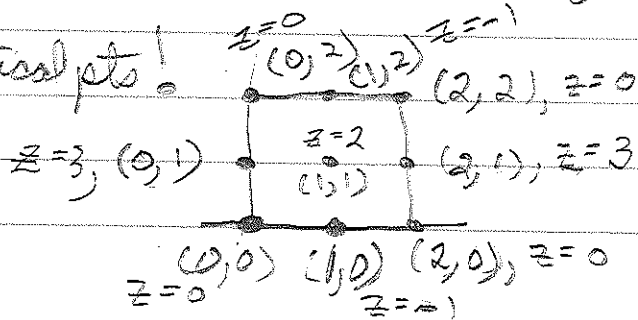
$\frac{df}{dx} = 2x - 2 = 0 \implies x = 1$ , so critical pt at  $(1,0)$

End pts also critical. Here  $(0,0), (2,0)$ .

d)  $y = 2, 0 \leq x \leq 2$  Work similar to above!  
critical pts  $(0,2), (1,2), (2,2)$ .

3.

9 critical pts!



max value 3 at  $(0,1), (2,1)$

min value -1 at  $(1,2), (1,0)$