Today we will study
- An introduction to hypothesis tests
- How to state a null hypothesis and alternative hypothesis
- How to identify type I and type II errors and interpret the level of significance
- How to make and interpret a decision based on the results of a statistical test
- The general steps for a hypothesis test using $P$-values

### Stating a Hypothesis

**DEFINITION**

1. A null hypothesis is a statistical hypothesis to be tested, denoted $H_0$. Must contain the condition of equality, such as $\leq$, $=$, $\geq$.

2. The alternative hypothesis is a statistical hypothesis to be considered as an alternative to the null hypothesis, denoted $H_a$. It is the complement of $H_0$. This is the statement that must be true if the null hypothesis is false. Contains the condition of inequality, such as $<$, $\neq$, $>$. 

### The Logic of Hypothesis Testing and Types of Errors

**Remarks**

No, matter which hypothesis $H_0$ or $H_a$ represents the claim, you always assume that $H_0$ is true.
When we perform a hypothesis test we make one of two possible decisions:

1. reject the null hypothesis
2. fail to reject the null hypothesis

**DEFINITION**

A type I error: rejecting the null hypothesis when it is in fact true.

A type II error: Not rejecting the null hypothesis when it is in fact false.

<table>
<thead>
<tr>
<th>Decision</th>
<th>Truth of $H_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$ is true</td>
<td>Correct decision</td>
</tr>
<tr>
<td>$H_0$ is false</td>
<td>Type II error</td>
</tr>
</tbody>
</table>

Compare with the legal system used in the United States.

<table>
<thead>
<tr>
<th>Verdict</th>
<th>Truth About Defendant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not Guilty</td>
<td>Justice</td>
</tr>
<tr>
<td>Guilty</td>
<td>Type I error</td>
</tr>
<tr>
<td></td>
<td>Justice</td>
</tr>
</tbody>
</table>
The test statistic is a value computed from the sample data that is used in making the decision about the rejection of the null hypothesis. The test statistic converts the sample statistic to a z-score with the assumption that the null hypothesis is true.

**DEFINITION**

In a hypothesis test, the level of significance is your maximum allowable probability of making a type I error, denoted \( \alpha \).

The probability of a type II error is denoted by \( \beta \).

Thus, the level of significance is the probability that the test statistic will fall in a (critical) region, that will cause us to reject the null hypothesis when the null hypothesis is actually true.

**Statistical Tests and \( P \)-values**

**DEFINITION**

If the null hypothesis is true, a \( P \)-value (or probability value) of a hypothesis test is the probability of obtaining a sample statistic with a value as extreme or more extreme than the one determined from the sample data.

Remarks

1. By extreme we mean: far from what we would expect to observe if the null hypothesis is true.
2. In other words, a small \( P \)-value indicates that observation of the test statistic would be unlikely if the null hypothesis is true.
   The lower the \( P \)-value, the more evidence there is in favor of rejecting the null hypothesis.
   A small \( P \)-value indicates a rare occurrence. (Not proof that \( H_0 \) is false)
3. The \( P \)-value depends on the type of hypothesis test. There are three types left-tailed, right-tailed and two-tailed.

**Making a Decision and Interpreting the Decision**

**Decision Rule Based on \( P \)-value**

To use a \( P \)-value to make a conclusion in a hypothesis test, compare the \( P \)-value with \( \alpha \)

1. If \( P \leq \alpha \), then reject \( H_0 \).
2. If \( P > \alpha \), then fail to reject \( H_0 \).

Remarks

1. Failing to reject \( H_0 \) does not mean that you have accepted \( H_0 \) as true
2. If you wish to support a claim, state it so that it is \( H_a \)
3. If you wish to disprove a claim, then state it as \( H_0 \)
### Claim Table

<table>
<thead>
<tr>
<th>Decision</th>
<th>Claim is $H_0$</th>
<th>Claim is $H_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reject $H_0$.</td>
<td>There is enough evidence to reject the claim</td>
<td>There is enough evidence to support the claim</td>
</tr>
<tr>
<td>Fail to Reject $H_0$.</td>
<td>There is <strong>Not</strong> enough evidence to reject the claim</td>
<td>There is <strong>Not</strong> enough evidence to support the claim</td>
</tr>
</tbody>
</table>

### Steps for Hypothesis Testing

1. State the claim. Identify the null hypothesis and alternative hypothesis.
2. Specify the level of significance, $\alpha$. \((\text{will be provided})\)
3. Determine the standardized sampling distribution and sketch its graph.
4. Calculate the test statistic and its standardized value (add to graph) - NEXT CLASS
5. Find the $P$-value.
   - *left-tailed test*: $P = \text{(Area in left tail)}$
   - *right-tailed test*: $P = \text{(Area in right tail)}$
   - *two-tailed test*: $P = 2\text{(Area in tail of test statistic)}$
6. Use the decision rule.
   - If $P \leq \alpha$, then reject $H_0$
   - If $P > \alpha$, then fail to reject $H_0$
7. Write a statement to interpret the decision in the context of the original claim.